

1. (a) **(3 points)** If $f(x, y) = x^2 - 5y$, $h(t) = t^2$ and $g(x, y) = 5x - y^2$, compute $f(h(2), g(1, 1))$ and $g(h(2), f(1, 1))$.
- (b) **(3 points)** Sketch and shade the domain of the function $f(x, y) = \sqrt{x(y^2 - x)}$. Use dotted lines to indicate portions of the boundary that are not included and solid lines to indicate portions of the boundary that are included.
2. (a) **(3 points)** Compute

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\tan 2(x^2 + y^2) + 3 \sin(x^2 + y^2)}{(x^2 + y^2)}.$$

- (b) **(3 points)** Show that

$$\lim_{(x,y) \rightarrow (1,2)} \frac{y - 2}{x - 1}$$

does not exist.

3. (a) **(3 points)** Find a point P at which the function $f(x, y) = x^2y$ has a local linear approximation $L(x, y) = 4y - 4x + 8$.
- (b) Determine dw for $w = \sqrt{x} + \sqrt{y} + \sqrt{z}$.
4. (a) **(3 points)** Suppose $w = xy + yz$, $y = \sin x$, $z = e^x$. Use a chain rule to find $\frac{dw}{dx}$.
- (b) **(3 points)** Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ for $ye^x - 5 \sin 3z = 3z$.
5. (a) **(3 points)** Given that $f_x(-5, 1) = -3$, $f_y(-5, 1) = 2$, find the directional derivative of f at the point $P(-5, 1)$ in the direction from P to $Q(-4, 3)$.
- (b) **(3 points)** Find a unit vector in the direction in which the functions $f(x, y) = 4e^{xy} \sin z$ decreases most rapidly at the point $P(0, 1, \frac{\pi}{3})$ and find the rate of change of f at P in that direction.