

1. (a) **(2 points)** Transform the equation $r = \csc \theta \cot \theta$ into rectangular coordinates and sketch its graph.
 - (b) **(3 points)** Transform the polar equation $x^2 + y^2 + 4y = 0$ into a polar equation of the form $r = f(\theta)$ and sketch its graph.
 - (c) **(2 points each)** Sketch the graphs of the following polar equations
 - i. $r = \cos \theta, \frac{\pi}{2} \leq \theta \leq \pi$
 - ii. $r = 1 + \cos \theta, -\frac{\pi}{2} \leq \theta \leq \pi$
 - iii. $r = 2 \sec \theta, 0 \leq \theta \leq \frac{\pi}{4}$.
2. (a) **(3 points)** Find $\frac{dy}{dx}$ at $t = \frac{\pi}{6}$ for the parametric curve $x = t + \cos t$, $y = 1 - \sin t$.
 - (b) **(2 points each)**
 - i. Show that the arc length of the cardioid $r = 1 + \cos \theta$ is given by $L = \sqrt{2} \int_0^{2\pi} \sqrt{1 + \cos \theta} d\theta$.
 - ii. Using the identity $1 + \cos \theta = 2 \cos^2 \frac{\theta}{2}$ show how to compute the arc length of the cardioid.
 - (c) **(4 points)** Find the equation of the tangent line to the graph of the curve $r = \frac{1}{\theta}$ at $\theta = \frac{\pi}{2}$.
3. (a) **(2 points)** Set up an integral to compute the area of the region in the first quadrant within the cardioid $r = 1 + \sin \theta$.
 - (b) **(3 points each)**
 - i. Find all points of intersection of the two cardioids $r = 1 + \cos \theta$ and $r = 3(1 - \cos \theta)$.
 - ii. Give the common area between the two cardioids in terms of integrals.