1. In each part find the equation of the sphere with center $(2,-1,-3)$ and satisfying the given conditions.

   (a) **(2 points)** Tangent to the xy-plane
   Radius of the sphere is $|-3| = 3$
   Therefore, equation of the sphere is

   $$(x-2)^2 + (y+1)^2 + (z+3)^2 = 9$$

   or

   $$x^2 + y^2 + z^2 + 2y - 4x + 6z + 5 = 0.$$

   (b) **(2 points)** Tangent to the x-axis
   Radius of the sphere is $\sqrt{(-1)^2 + (-3)^2} = \sqrt{10}$
   Therefore, equation of the sphere is

   $$(x-2)^2 + (y+1)^2 + (z+3)^2 = 10$$

   or

   $$x^2 + y^2 + z^2 + 2y - 4x + 6z + 3 = 0.$$

   (c) **(2 points)** Passes through the origin
   Radius of the sphere is $\sqrt{(2)^2 + (-1)^2 + (-3)^2} = \sqrt{14}$
   Therefore, equation of the sphere is

   $$(x-2)^2 + (y+1)^2 + (z+3)^2 = 14$$

   or

   $$x^2 + y^2 + z^2 + 2y - 4x + 6z = 0.$$

2. A parallelogram has $\mathbf{i} - \mathbf{j} + \mathbf{k}, \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ as adjacent sides. Find

   (a) **(2 points)** Its area.

   Area $= \|(\mathbf{i} - \mathbf{j} + \mathbf{k}) \times (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})\| = \left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 1 & -3 & 2 \end{vmatrix} \right\| = \|\mathbf{i} - \mathbf{j} - 2\mathbf{k}\| =$
   $\sqrt{6}$

   (b) **(2 points)** The lengths of its heights.
   The parallelogram has two heights. If $\mathbf{i} - \mathbf{j} + \mathbf{k}$ is the base, the height is $\frac{\sqrt{6}}{\|\mathbf{i}-\mathbf{j}+\mathbf{k}\|} = \sqrt{2}$.
   If $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ is the base, the height is $\frac{\sqrt{6}}{\|\mathbf{i}-3\mathbf{j}+2\mathbf{k}\|} = \sqrt{\frac{3}{7}}$.

   (c) **(2 points)** The lengths of its diagonals.
   One diagonal is $(\mathbf{i} - \mathbf{j} + \mathbf{k}) + (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = 3\mathbf{k} - 4\mathbf{j} + 2\mathbf{i}$. Its length is $\sqrt{29}$.
   The other diagonal is $(\mathbf{i} - \mathbf{j} + \mathbf{k}) - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = 2\mathbf{j} - \mathbf{k}$. Its length is $\sqrt{5}$

3. Given the points $P(-3,1,2)$, $A(1,1,0)$, $B(-2,3,-4)$ find

   (a) **(2 points)** $\text{Proj}_{\overrightarrow{AB}}\overrightarrow{AP}$

   $\overrightarrow{AB} = \langle -3,1,-4\rangle$, $\overrightarrow{AP} = \langle -4,0,2\rangle$. $\text{Proj}_{\overrightarrow{AB}}\overrightarrow{AP} = \frac{\overrightarrow{AB}\cdot\overrightarrow{AP}}{\left\|\overrightarrow{AB}\right\|^2}\overrightarrow{AB} =$ $\frac{2}{7}\langle -3,1,-4\rangle$.

   (b) **(2 points)** The component of $\overrightarrow{AP}$ orthogonal to $\overrightarrow{AB}$.

   $\mathbf{v} = \overrightarrow{AP} - \text{Proj}_{\overrightarrow{AB}}\overrightarrow{AP} = \langle -4,0,2\rangle - \frac{2}{7}\langle -3,1,-4\rangle = \left\langle -\frac{22}{7}, -\frac{2}{7}, \frac{22}{7}\right\rangle$

   (c) **(2 points)** The distance from the point P to the line through A, B.

   Distance $= \|\mathbf{v}\| = \frac{2}{7}\sqrt{243}$.

4. Consider the parallelepipped with adjacent edges $\mathbf{u} = \langle 2,2,1\rangle$, $\mathbf{v} = \langle 1,1,2\rangle$, $\mathbf{w} = \langle 1,3,3\rangle$.

   (a) **(2 points)** Find the volume

   $$\text{Volume} = \left\| \begin{array}{ccc} 2 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{array} \right\| = |-6| = 6$$

   (b) **(2 points)** Find the area of the face determined by $\mathbf{u}$ and $\mathbf{w}$.

   $$\text{Area} = \|\mathbf{u}\times\mathbf{w}\| = \left\| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 1 \\ 1 & 3 & 3 \end{vmatrix} \right\| = \|3\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}\| = \sqrt{16+25+9}$$

   $= \sqrt{50} = 7.0711$

   (c) **(2 points)** Find the angle between $\mathbf{v}$ and the face determined by $\mathbf{u}$ and $\mathbf{w}$.

   Length of perpindicular from $\mathbf{v}$ to the base $\mathbf{u},\mathbf{w}$ $= \frac{6}{\sqrt{50}}$. Therefore, if θ is the required angle, $\tan\theta = \frac{6}{\sqrt{50}} / \|\mathbf{v}\| = \frac{6}{\sqrt{50}\sqrt{6}} = \frac{\sqrt{3}}{5}$.

5. Find parametric equations of the line through the point $(5,0,-2)$ that is parallel to the planes $x - 4y + 2z = 0$ and $2x + 3y - z + 1 = 0$.

   The required line is parallel to the line of intersection of the two planes. Therefore, its direction is

   $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 2 & 3 & -1 \end{vmatrix} = -2\mathbf{i} + 5\mathbf{j} + 11\mathbf{k}$$

   Equation of the required plane is

   $$-2(x-5) + 5y + 11(z+2) = 0$$

   or

   $$-2x + 5y + 11z + 32 = 0.$$

6. Find the equation of the plane through the origin that is parallel to the plane $4x - 2y + 7z + 12 = 0$.

Normal to the plane is $\langle 4, -2, 7 \rangle$. Equation of the plane: $4x - 2y + 7z = 0$.