1. **(4pts)** Find the equation of the sphere with center at $(2, 3, -1)$ That passes through the point $(4, -1, 1)$.

   Radius $= \sqrt{(4-2)^2 + (-1-3)^2 + (1+1)^2} = \sqrt{24}$.

   Therefore, equation is : $(x-2)^2 + (y+1)^2 + (z-1)^2 = 24$
   or $x^2 + y^2 + z^2 + 2y - 4x - 2z = 18$.

2. **(4pts)** Find the point C on the line segment joining $A(2, 2, 1)$ to $B(3, -1, 2)$ that divides it in the ratio $2 : 3$

   The point is $C = \frac{1}{3}A + \frac{2}{3}B = \frac{1}{3}(2, 2, 1) + \frac{2}{3}(3, -1, 2) = \left(\frac{8}{3}, 0, \frac{5}{3}\right)$.

3. **(4pts)** Find the vector component of $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ along $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and the vector component of $\mathbf{v}$ orthogonal to $\mathbf{b}$.

   $\text{Proj}_{\mathbf{b}}\mathbf{v} = \frac{\mathbf{v}\cdot\mathbf{b}}{\|\mathbf{b}\|^2}\mathbf{b} = \frac{6}{9}(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = \frac{2}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} + \frac{4}{3}\mathbf{k}$.

   Orthogonal component $= (2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) - \left(\frac{2}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} + \frac{4}{3}\mathbf{k}\right) = \frac{4}{3}\mathbf{i} - \frac{7}{3}\mathbf{j} + \frac{5}{3}\mathbf{k}$.

4. **(4pts)** Find the area of the triangle ABC, where $A = (2, -2, 1)$, $B = (3, -1, 2)$, $C = (3, -2, 3)$.

   Area of the triangle $= \frac{1}{2}\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\| = \left\|\left|\begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{matrix}\right|\right\| = \frac{1}{2}\|2\mathbf{i} - \mathbf{k} - \mathbf{j}\| = \frac{\sqrt{6}}{2}$.

5. **(4pts)** Find the parametric equations of the line that contains the point $P(0, 2, 1)$ and intersects the line $L : x = 2t,\ y = 1 - 2t,\ z = 3 - t$ at a right angle.

   Assume the point on the line is $C(2t, 1-2t, 3-t)$. The vector $\overrightarrow{PC}$ is orthogonal to the line L. Therefore,

   $$\overrightarrow{PC} \cdot \langle 2, -2, -1\rangle = \langle -2t, 1+2t, -2+t\rangle \cdot \langle 2, -2, -1\rangle = 0.$$

   This gives the equation $-4t - 2 - 4t + 2 - t = -9t = 0$. Hence $t = 0$ and $\overrightarrow{PC} = \langle 0, 1, -2\rangle$.

   Equations of the line are $x = 0,\ y = 2 - t,\ z = 1 + 2t$.

6. **(4pts)** Find parametric equations of the line through the point $(5, 0, -2)$ that is parallel to the planes $x - 4y + 2z = 0$ and $2x + 3y - z + 1 = 0$.

   The line is parallel to the line of intersection of the two planes. Therefore, it direction is

   $$\left|\begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 2 & 3 & -1 \end{matrix}\right| = -2i + 5j + 11k.$$

   Equations of the line are $x = 5 - 2t,\ y = 5t,\ z = -2 + 11t$.

7. **4pts)** Find the distance between the point $(2, 3, -1)$ and the plane $2x + y + z = 0$.

   $$D = \frac{|2 \times 2 + 1 \times 3 - 1 \times 1|}{\sqrt{4 + 1 + 1}} = \sqrt{6}$$

8. (**4pts)** Locate the point of intersection of the plane $2x + y - z = 0$ and the line through $(3, 1, 0)$ that is perpendicular to the plane.

The line is parallel to the normal to the plane. Its equation is $x = 3 + 2t,\ y = 1 + t,\ z = -t$. It intersects the plane when $2(3 + 2t) + (1 + t) + t = 6t + 7 = 0$. This gives $t = -\frac{7}{6}$. Insert this value of t in the equation of the line to get the point of intersection $\left(\frac{2}{3}, -\frac{1}{6}, \frac{7}{6}\right)$.

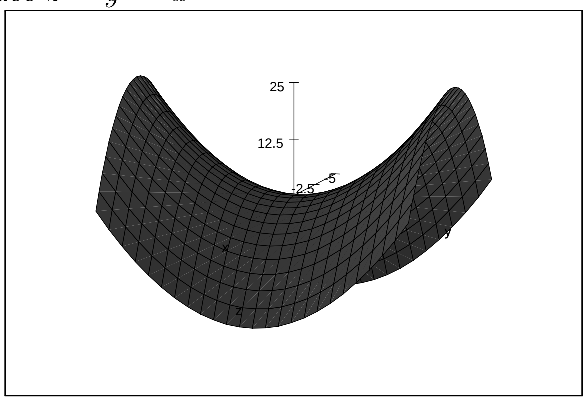
9. (**4pts)** Find the points of intersection of the line $x = 2t,\ y = 1 - t,\ z = 2 - 3t$ and the coordinate planes.

xy-intercept: put $z = 0$, this gives $t = \frac{2}{3}$. Point of intersection is $\left(\frac{4}{3}, \frac{1}{3}, 0\right)$.

yz-intercept. Put $x = 0$, this gives $t = 0$. Point of itnersection is $(0, 1, 2)$

zx-intercept. Put $y = 0$, this gives $t = 1$. Point of itnersection is $(2, 0, -1)$

10. (**4pts)** Sketch the surface $z = y^2 - x^2$



. What are the traces of this surface in the planes $z = 1,\ z = 0,\ z = -1$?

Trace in the plane $z = 1$ is the hyperbola $y^2 - x^2 = 1$. Trace in the plane $z = 0$ is the two lines $y = \pm x$. Trace in the plane $z = -1$ is the hyperbola $x^2 - y^2 = 1$.