

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 311
Exam # 3
May, 8, 2006

NAME:

ID#:

1. **(5points)** State and prove the Mean Value Theorem

2. (4points) Calculate

$$\lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^3}$$

3. (4points) Use the Mean Value Theorem to prove that

$$\sqrt{1+x} \leq 1 + \frac{x}{2}.$$

4. **(4points)** Show that if a function $f : D \subset \mathbb{R} \rightarrow \mathbb{R}$ has a bounded derivative, then it is uniformly continuous.

5. **(4points)** Show that if a function f is differentiable at c , then it is continuous at c .

6. **(4points)** Define the function $f : \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = x^2$ if x is rational and $f(x) = 0$ if x is irrational. Is f differentiable at 0?

7. **(5points)** State and prove the fundamental lemma of differentiation.