1. Show that $f(x) = \frac{1}{x}$ is uniformly continuous on $[1, \infty)$.

2. Show that if $f, g : \mathbb{R} \to \mathbb{R}$ are uniformly continuous, then so is $f + g$.

3. Show that if f is continuous on (a, b) and has finite limits as $x \to a^+$ and $x \to b^-$ then f is uniformly continuous on $[a, b]$.

4. Show that the family $\mathcal{F} = \left\{ \left(\frac{1}{n+2}, \frac{1}{n} \right) \right\}_{n=1}^{\infty}$ is an open cover for $J = \left(0, \frac{1}{2}\right)$, but no finite subfamily of $\mathcal{F}$ covers J.

5. Suppose f is differentiable on $I = (a, b)$. Compute

$$\lim_{h \to 0} \frac{f(x + \alpha h) - f(x + \beta h)}{h}.$$

6. State and prove the chain rule.

7. Show that if f is continuous on $[a, b]$, differentaible on (a, b) and $f'(x) > 0$ for all $x \in (a, b)$, then f is increasing on $[a, b]$.

8. Calculate

$$\lim_{x \to 0} \frac{\tan\left(\sin^{-1} x\right) - \sin\left(\tan^{-1} x\right)}{x^3}, \quad \lim_{x \to \infty} \left(1 + \frac{\alpha}{x}\right)^{\beta x}.$$

9. Show that the function $f(x) = \begin{cases} e^{1/x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ has the property that $f^{(n)}(0) = 0$ for every positive integer n.

10. Suppose f is continuous on $[a, b]$, differentaible on (a, b) and f is not 1:1 on $[a, b]$. Show that there is a $c \in (a, b)$ such that $f'(c) = 0$.