

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 260
Final Exam, Spring 052
May 31, 2006

NAME:

ID#:

SHOW ALL YOUR WORK

1. **(10points)** Find a general solution of the differential equations

(a) $xy' = 2y + x^3 \cos x.$

(b) $xy' + 6y = 3xy^{4/3}.$

2. (a) **(10points)** Find a general solution of the differential equation

$$y'' + 4y = 3x \cos 2x.$$

- (b) **(4points)** Find a homogeneous differential equation with constant coefficients and minimal order for which $xe^{-2x} \cos 3x$ and e^{-2x} are two of its solutions.

3. **(6points)** Find a basis for the solution space of the homogeneous linear system

$$3x_1 + 6x_2 - x_3 - 5x_4 + 5x_5 = 0$$

$$2x_1 + 4x_2 - x_3 - 3x_4 + 2x_5 = 0$$

$$3x_1 + 6x_2 - 2x_3 - 4x_4 + x_5 = 0$$

4. **(10points)** Find a matrix M and a diagonal matrix D such that the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

can be written as $A = M^{-1}DM$.

5. (a) **(8points)** Find a general solution of the linear system

$$x' = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} x.$$

- (b) **(2points)** Find the solution satisfying the initial condition $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

6. (a) **(7points)** Find a general solution of the linear system

$$x' = \begin{bmatrix} 1 & -4 \\ 4 & 9 \end{bmatrix} x.$$

(b) **(3points)** Find the solution satisfying the initial condition $x(0) = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ and compute the solution at $t = 4$.

7. (a) **(4points)** Show that the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable.
- (b) **(2points)** Write $A = I + B$. Show that $B^2 = 0$.
- (c) **(4points)** Show that $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$.
- (d) **(Extra Credit: 4 points)** Using the power series for $\cos x$ compute $\cos(At)$ in simplest form.