KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 260
Exam # 3
May 6, 2006

NAME:	ID#:

SHOW ALL YOUR WORK

1. **(5points)** In the vector space $\mathbb{R}^4$ define the subsets

   $W_1 =$ the set of all vectors $v = (x_1, x_2, x_3, x_4)$ in $\mathbb{R}^4$ such that $x_1 + x_2 = x_3 - x_4$,

   $W_2 =$ the set of all vectors $v = (x_1, x_2, x_3, x_4)$ in $\mathbb{R}^4$ such that $x_1 x_2 = x_3 x_4$.

   Determine (giving reasons) whether or not W_1 and W_2 are subspaces of $\mathbb{R}^4$.

2. **(5points)** Is the vector $v = -7, 7, 11)$ in the span of the vectors $v_1 = (1, 2, 1), v_2 = (-4, -1, 7), v_3 = (-3, 1, 3)$?. If so, express v as a linear combination of v_1, v_2, v_3.

3. **(5points)** If v_1, v_2, v_3 are linearly independent vectors in a vector space v, show that the vectors $u_1 = v_2 + v_3,\ u_2 = v_1 + v_3, u_3 = v_1 + v_2$ are also linearly independent..

4. **(5points)** Find a basis for the solution space of the homogeneous linear system

$$\begin{aligned} x_1 - 3x_2 - 9x_3 - 5x_4 &= 0 \\ 2x_1 + x_2 - 4x_3 + 11x_4 &= 0 \\ x_1 + 3x_2 + 3x_3 + 13x_4 &= 0 \end{aligned}$$

5. **(5points)** Determine (with reasons) whether or not the functions $f(x) = x, g(x) = |x|$ are linearly independent on:

   (a) the real line,

   (b) the interval $I = (0, \infty)$.

6. **(5points)** Suppose $1, r, s$ are all distinct numbers. Show that

$$W(e^x, e^{rx}, e^{sx}) = e^{(1+r+s)x} \begin{vmatrix} 1 & 1 & 1 \\ 1 & r & s \\ 1 & r^2 & s^2 \end{vmatrix}$$

   and hence, show that the functions e^x, e^{rx}, e^{sx} are linearly independent on the real line.