

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 260
Exam # 2
April, 8, 2006

NAME:

ID#:

SHOW ALL YOUR WORK

1. **(6points)** Write the following matrices in (a) Row Echlon Form, and (b) Reduced Row Echlon Form. Show details of your work.

$$(i) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (ii) \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 3 \\ 3 & 1 \end{bmatrix}, \quad (iii) \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

2. Suppose A, B are $n \times n$ matrices with $\det A = -5$ and $\det B = 3$. Compute:

- (i) **(2pts)** $\det(2A)$, (ii) **(1pt)** $\det(AB)$, (iii) **(2pts)** $\det(A(AB)^{-1})$, (iv) **(1pt)** $\det(A^T A)$

3. **(6points)** Write the augmented matrix for the system

$$2x + 5y + 12z = 6$$

$$3x + y + 5z = 12$$

$$5x + 8y + 21z = 17$$

then solve the system by Gauss Elimination.

4. **(6points)** Find A^{-1} by writing the augmented matrix $[A \mid I]$ and then using the row

reduction operations, where $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$.

5. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

- (a) **(2points)** Show that $A^2 = A + I$.
- (b) **(2points)** Use part (a) to show that $A^3 = 2A + I$ and $A^4 = 3A + I$.
- (c) **(1points)** Express A^n in terms of A and I .
- (d) **(1points)** Express A^{-1} in terms of A and I .
- (e) **(2points extra credit)** Find the matrix B if

$$B = I + \sum_{n=1}^{\infty} \frac{A^n}{n!}.$$

(You may take $A^0 = I$.)