KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 260-04
Quiz # 5
January 15, 2007

1. Let A be the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Show that A is not diagonalizable.

A has only one eigenvaule $\lambda = 1$. The corresponding eigenvectors:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Since A has only two eigenvectors, it is not diagonalizable.

2. Suppose A is the matrix

$$A = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix},$$

where $0 < p < 1$ and $0 < q < 1$. Show that $\lambda = 1$ is an eigenvalue for A. What is the other eigenvalue? What are the corresponding eigenvectors.

$$\begin{aligned} |A - \lambda I| &= \begin{vmatrix} p-\lambda & 1-p \\ 1-q & q-\lambda \end{vmatrix} \\ &= \begin{vmatrix} 1-\lambda & 1-p \\ 1-\lambda & q-\lambda \end{vmatrix} \text{ (by adding the first two columns)} \\ &= (1-\lambda)\begin{vmatrix} 1 & 1-p \\ 1 & q-\lambda \end{vmatrix} \\ &= (1-\lambda)(q-\lambda+p-1) = 0 \end{aligned}$$

gives $\lambda = 1$ or $\lambda = p + q - 1$.

For $\lambda = 1$: the row reduction operations give

$$\begin{bmatrix} p-1 & 1-p \\ 1-q & q-1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

therefore, the corresponding eigenvector is

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

for $\lambda = p + q - 1$ the row reduction operations give

$$\begin{bmatrix} 1-q & 1-p \\ 1-q & 1-p \end{bmatrix} \to \begin{bmatrix} 1-q & 1-p \\ 0 & 0 \end{bmatrix}$$

which gives the eigenvector

$$\begin{bmatrix} p-1 \\ 1-q \end{bmatrix}.$$