ING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 260-04
Quiz # 4

Find a particular solution for each of the following differential equations

1. $y'' + 2y' - 3y = 2 + e^x$.

   Characteristic polynomial: $r^2 + 2r - 3$, roots: $r = -3,\ r = 1$. Solution of the homogeneous equatoin: $y = e^{-3x}, y = e^x$. Killer of the rhs: $D(D-1)$. Applying the killer gives

$$D(D-1)^2(D+3)y = 0$$

   with characteristic roots $0, 1, 1, -3$ and corresponding solutions $y = 1, y = e^x, y = xe^x, y = e^{-3x}$. Eleminating the common solutions gives a particular solution of the form $y_p = \alpha + \beta x e^x$. Substituting in the original differential equation gives

$$\begin{aligned}(D+3)(D-1)(\alpha + \beta x e^x) &= 2 + e^x \\ (D+3)(-\alpha + \beta e^x) &= 2 + e^x \\ -3\alpha + 4\beta e^x &= 2 + e^x.\end{aligned}$$

   Therefore, $\alpha = -2/3$ and $\beta = 1/4$. Thus $y_p = -2/3 + xe^x/4$.

2. $y'' + y = \tan x$.

   Since $\tan x$ has no killer, we have to use the method of variation of the parameter. The solutions of the homogeneous equation are $\cos x,\ \sin x$.

$$\begin{aligned}u_1' &= \frac{\begin{vmatrix} 0 & \sin x \\ \tan x & \cos x \end{vmatrix}}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}} = -\sin x \tan x \\ &= -\frac{\sin^2 x}{\cos x} = \cos x - \sec x. \\ u_1 &= \sin x - \ln|\sec x + \tan x|.\end{aligned}$$

$$\begin{aligned}u_2' &= \begin{vmatrix} \cos x & 0 \\ -\sin x & \tan x \end{vmatrix} = \sin x \\ u_2 &= -\cos x.\end{aligned}$$

   Thus

$$\begin{aligned}y_p &= \cos x(\sin x - \ln|\sec x + \tan x|) - \cos x \sin x \\ &= -\cos x \ln|\sec x + \tan x|.\end{aligned}$$