

ING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 260-04
Quiz # 3

1. Express the vector $\mathbf{t} = (12, 1, 9)$ as a linear combination of the vectors $\mathbf{u} = (1, -2, 2)$ and $\mathbf{v} = (3, -1, 3)$

Ans: We need to find a solution for the augmented system

$$\begin{bmatrix} 1 & 3 & 12 \\ -2 & -1 & 1 \\ 2 & 3 & 9 \end{bmatrix}.$$

The echlon form is

$$\begin{bmatrix} 1 & 3 & 12 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix},$$

which gives the solutions $c_2 = 5, c_1 = -3$. Therefore,

$$\mathbf{t} = -3\mathbf{u} + 5\mathbf{v}.$$

2. Determine, with reason, whether or not the following subsets are subspaces of their respective spaces:

(a) **(3point)** $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : 2x_1 + 3x_2 = 0\}$.

(b) **(3point)** $W = \{(x_1, x_2) \in \mathbb{R}^2 : |x_1| = x_2\}$.

Ans: (a) Let $\mathbf{x} = (x_1, x_2, x_3), \mathbf{y} = (y_1, y_2, y_3) \in W$ and let $c \in \mathbb{R}$. Then $2x_1 + 3x_2 = 0$ and $2y_1 + 3y_2 = 0$. Then

$$\mathbf{x} + c\mathbf{y} = (x_1 + cy_1, x_2 + cy_2, x_3 + cy_3)$$

and

$$\begin{aligned} 2(x_1 + cy_1) + 3(x_2 + cy_2) &= (2x_1 + 3x_2) + c(2y_1 + 3y_2) \\ &= 0 + c0 = 0. \end{aligned}$$

Thus $\mathbf{x} + c\mathbf{y} \in W$ and W is a subspace of \mathbb{R}^3 .

(b) Let $\mathbf{u} = (-1, 1)$ and $c = -1$. Then $u \in W$ but $cu = (1, -1)$ does not satisfy the condition $|x_1| = x_2$. Therefore, W is not a subspace.