

1. What is the minimum and maximum possible number of free variables for the following systems:

(a) A system of 2 equations and 5 variables: min: 3 ;max: 4

(b) A system of 3 equations and 3 variables: min: 0 ;max: 2

(c) A system of 3 equations and 2 variables: min: 0 ;max: 1

2. The following linear system is written in echlon form:

$$\begin{aligned}x_1 - 5x_2 + 2x_3 - 7x_4 + 11x_5 &= 1 \\x_2 - 13x_3 + 3x_4 - 7x_5 &= -1 \\x_4 - 5x_5 &= 2.\end{aligned}$$

(a) Write the augmented matrix for the system.

$$\left[ \begin{array}{cccccc} 1 & -5 & 2 & -7 & 11 & 1 \\ 0 & 1 & -13 & 3 & -7 & -1 \\ 0 & 0 & 0 & 1 & -5 & 2 \end{array} \right]$$

(b) Identify the leading variables.  $x_1, x_2, x_4$

(c) Identify the free variables.  $x_3, x_5$

(d) Solve the system by back substitution.

$$\begin{aligned}x_5 &= s \\x_4 &= 2 + 5s \\x_3 &= r \\x_2 &= -7 + 13r - 8s \\x_1 &= -20 + 63r - 16s\end{aligned}$$

(e) Show that all solutions of the system can be written in the form  $\mathbf{b}_0 + r\mathbf{b}_1 + s\mathbf{b}_2$ , where  $r$  and  $s$  are arbitrary real numbers and  $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2$  are certain vectors (you should give explicitly the vectors  $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2$ ).

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -20 \\ -7 \\ 0 \\ 2 \\ 0 \end{bmatrix} + r \begin{bmatrix} 63 \\ 13 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -16 \\ -8 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$