1. For the differential equation $x\frac{dy}{dx} + 3y = 2x^5$

   (a) Verify that $y = \frac{1}{4}x^5 + Cx^{-3}$ is a general solution.
   $y' = \frac{5}{4}x^4 - 3Cx^{-4}$. $\quad xy' = \frac{5}{4}x^5 - 3Cx^{-3}$.
   LHS $=\ xy' + 3y = \frac{5}{4}x^5 - 3Cx^{-3} + \frac{3}{4}x^5 + 3Cx^{-3} = 2x^5$ =RHS.

   (b) Find a particular solution satisfying $y(2) = 1$.
   At $x = 2$, $y = 1$. Therefore, $1 = \frac{1}{4} \times 32 + \frac{C}{8} = 8 + \frac{C}{8}$. This gives $C = -56$. The particular solution is $y = \frac{1}{4}x^5 - 56x^{-3}$

2. Solve the initial value problem $\frac{dy}{dx} = \frac{10}{x^2+1}$, $y(1) = 2$.

   $y = \int \frac{10}{x^2+1}dx = 10\tan^{-1}x + C$. At $x = 1$, $y = 2$. Thus $2 = 10\tan^{-1}1 + C = \frac{5\pi}{2} + C$. This gives $C = 2 - \frac{5\pi}{2}$. The particular solution is $y = 10\tan^{-1} + 2 - \frac{5\pi}{2}$.

3. **(2points)** Extra Credit In the Swimmers Problem, if the velocity v_R of flow of the water is represented by a cosine function, find an explicit representation of v_R as a function of x and then solve the differential equation $\frac{dy}{dx} = \frac{v_R}{v_S}$, where v_S is the swimmer's velocity (assumed to be constant).

   $v_R = \frac{v_0}{2}\left(1 + \cos\frac{\pi x}{a}\right)$. The differential equation is

   $$\frac{dy}{dx} = \frac{v_0}{2v_S}\left(1 + \cos\frac{\pi x}{a}\right).$$

   Solving, we obtain

   $$y = \frac{v_0}{2v_S}\left(x + \frac{a}{\pi}\sin\frac{\pi x}{a}\right) + C$$

   Using the initial condition $y(-a) = 0$ gives $C = \frac{v_0}{2v_S}a$. Hence, we have the solution

   $$y = \frac{v_0}{2v_S}\left(x + a + \frac{a}{\pi}\sin\frac{\pi x}{a}\right).$$

   The swimmer lands at $y(a) = \frac{v_0}{v_S}a$.