

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 202-11

Quiz # 3
December 16, 2009

1. For the differential equation $y'' + 4y' + 4y = 4 \cos 2x + 3 \sin 2x - 8$, (i) find the solutions of the homogeneous equation and (ii) find an annihilator of lowest order for the right hand side.

Solution:

(i) The auxiliary equation $m^2 + 4m + 4 = 0$ has the repeated $m = -2$. The homogenous solutions then are: $y_1 = e^{2x}$ and $y_2 = xe^{2x}$.

(ii) The operator $(D^2 + 4)$ kills both $\cos 2x$ and $\sin 2x$ and the operator D kills 8. Therefore, the required annihilator is $D(D^2 + 4)$.

2. Two roots of a cubic auxiliary equation with real coefficients are $m_1 = \frac{1}{2}$ and $m_2 = 1 + 3i$. What is the corresponding linear homogeneous differential equation?

Solution:

The auxiliary equation can be written as

$$\begin{aligned} \left(m - \frac{1}{2}\right) (m - (1 + 3i)) (m - (1 - 3i)) &= 0 \\ \left(m - \frac{1}{2}\right) (m^2 - 2m + 10) &= 0 \\ (2m - 1) (m^2 - 2m + 10) &= 0 \\ 2m^3 - 5m^2 + 22m - 10 &= 0, \end{aligned}$$

from which we get the differential equation

$$2y''' - 5y'' + 22y' - 10y = 0.$$

Remark: observe that $(m - (\alpha + \beta i))(m - (\alpha - \beta i)) = m^2 - 2\alpha m + (\alpha^2 + \beta^2)$.