1. Solve the initial value problem $y^{1/2}\frac{dy}{dx}+y^{3/2}=1, \quad y(0)=4.$

   Solution

   Multiply both sides by $y^{-1/2}$:
$$\frac{dy}{dx}+y=y^{-1/2}.$$
   This is a Bernouli equation with $n=-1/2$. Use the substitution
$$\begin{aligned} u &= y^{1+1/2}=y^{3/2} \\ y &= u^{2/3} \\ y' &= \frac{2}{3}u^{-1/3}u'. \end{aligned}$$
   Then
$$\begin{aligned} \frac{2}{3}u^{-1/3}u'+u^{2/3} &= u^{-1/3} \\ u'+\frac{3}{2}u &= \frac{3}{2}. \end{aligned}$$
   The integrating factor is $e^{3/2x}$. We then proceed as follows
$$\begin{aligned} \left(e^{3/2x}u\right)' &= \frac{3}{2}e^{3/2x} \\ e^{3/2x}u &= e^{3/2x}+C \\ u &= 1+Ce^{-3/2x} \\ y^{3/2} &= 1+Ce^{-3/2x}. \end{aligned}$$
   At $x=0$, $y=4$, therefore,
$$\begin{aligned} 8 &= 1+C \\ C &= 7 \end{aligned}$$
   The solution of the differential equation is
$$y^{3/2}=1+7e^{-3/2x}.$$

2. Solve the differential equation $x\frac{dy}{dx}-y=x^2\sin x.$

   Solution

   This is a linear first order equation wnich can be written in standard form as
$$\frac{dy}{dx}-\frac{1}{x}y=x\sin x.$$
   The integrating factor is $e^{\int -1/x dx}=\frac{1}{x}$. We proceed as follows
$$\begin{aligned} \left(\frac{1}{x}y\right)' &= \sin x \\ \frac{1}{x}y &= -\cos x+C \\ y &= -x\cos x+Cx \end{aligned}$$
   defined on the interval $(0,\infty)$, say.