Quiz # 1

October 14, 2009

1. For the differential equation $x\frac{dy}{dx} + 3y = 2x^5$

   (a) Verify that $y = \frac{1}{4}x^5 + Cx^{-3}$ is a general solution.

   (b) Find a solution satisfying the initial condition $y(2) = 1$.

   a. We compute:

$$\begin{aligned}
\frac{dy}{dx} &= \frac{5}{4}x^4 - 3Cx^{-4}. \\
x\frac{dy}{dx} &= \frac{5}{4}x^5 - 3Cx^{-3}. \\
3y &= \frac{3}{4}x^5 + 3Cx^{-3}. \\
x\frac{dy}{dx} + 3y &= \frac{5}{4}x^5 - 3Cx^{-3} + \frac{3}{4}x^5 + 3Cx^{-3} \\
&= \frac{8}{4}x^5 = 2x^5.
\end{aligned}$$

   b. At $x = 2$, $y = 1$. Substituting in the general solution gives

$$\begin{aligned}
1 &= \frac{32}{4} + \frac{C}{8}. \\
-7 &= \frac{C}{8}. \\
C &= -56.
\end{aligned}$$

   Therefore, the required solution is

$$y = \frac{1}{4}x^5 - 56x^{-3}.$$

2. **(4points)** Determine a region in the xy-plane for which the differential equation $\frac{dy}{dx} = y^{2/3}$ would have a unique solution whose graph passes through a point (x_0, y_0) in the region.

$$\begin{aligned}
f(x, y) &= y^{2/3}, \\
\frac{\partial f(x, y)}{\partial y} &= \frac{2}{3}y^{-1/3}.
\end{aligned}$$

   f is continuous on the whole xy-plane while $\frac{\partial f}{\partial y}$ is continuous on any region that does not contain the x-axis ($y = 0$). Therefore, we may take the region R as

$$R = (-\infty, \infty) \times (0, \infty).$$