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A blowup result in a multidimensional semilinear thermoelastic system *

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Abstract

In this work, we consider a multidimensional semilinear system of thermoelasticity and show that the energy of any weak solution blows up in finite time if the initial energy is negative. This work generalizes earlier results in [5] and [8].

1 Introduction

In [8], we considered the one-dimensional Cauchy problem

$$\begin{gathered} u_{tt}(x,t) = au_{xx}(x,t) + b\theta_x(x,t) + |u(x,t)|^{\alpha-1}u(x,t) \\ c\theta_t(x,t) = k\theta_{xx}(x,t) + bu_{xt}(x,t), \quad x \in \mathbb{R}, t \geq 0 \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad \theta(x,0) = \theta(x), \quad x \in \mathbb{R}, \end{gathered}$$

where a, c, k are strictly positive constants, b is a nonzero constant, and $\alpha \geq \sqrt{1 + b^2/(ac)}$. We showed that any weak solution with negative initial energy blows up in finite time if u_0 and u_1 are cooperative ($\int u_0u_1 > 0$). This result was improved by Kirane and Tatar [5], where the authors studied a more general system by allowing gradient terms in both equations. To overcome the difficulty caused by these extra terms, they defined a functional which satisfies the conditions of a theorem by Kalantarov and Ladyzhenskaya [4]. Their result, when applied to the system in [8], omits the condition of cooperative initial data, however the condition on α remained (see relation 13 of [5]).

In [17], Racke and Wang discussed the propagation of singularities for systems of homogeneous thermoelasticity in one spatial dimension. They considered some linear and semilinear Cauchy problems and described the propagation of singularities, as well as, the distribution of regular domains if the initial data have different regularity in different parts of the real line.

Concerning global existence and asymptotic behavior of weak solutions, it is worth noting the work of Aassila [1], where a purely linear multidimensional

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system of inhomogeneous and anisotropic thermoelasticity, associated with nonlinear boundary conditions, has been studied. Under suitable requirements on the nonlinear terms at the boundary, the author proved a decay result. As he mentioned, his result extends the one in [14] to the nonlinear case. For results regarding the matter of existence, regularity, controllability, and long-time behavior of systems of thermoelasticity, we refer the reader to articles [2], [3], [12], [13], [15], [16], and [18].

In this paper we are concerned with the initial boundary value problem

$$\begin{gathered} u_{tt}(x,t) = \operatorname{div}(A(x)\nabla u(x,t)) + b(x)\cdot\nabla\theta(x,t) + D(x)\cdot\nabla u(x,t) \\ -m(x)u_t(x,t) + e^{\beta t}u(x,t)|u(x,t)|^{p-2}, \quad x\in\Omega, t>0 \\ c(x)\theta_t(x,t) = \operatorname{div}[K(x)\nabla\theta(x,t) + b(x)u_t(x,t)] + R(x)\cdot\nabla u(x,t) \\ u(x,t)=0, \quad \theta(x,0)=0, \quad x\in\partial\Omega, t\ge 0 \\ u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), \quad \theta(x,0)=\theta(x), \quad x\in\Omega, \end{gathered} \tag{1.1}$$

where b, D, R are “function entry” n-component real vectors; c, m are functions; A, K are $n\times n$ “function entry” matrices such that A is symmetric; $b\neq 0$, $p>2, \beta>0$; and Ω is a bounded domain of $\mathbb{R}^n$ $(n\ge 1)$, with a smooth boundary $\partial\Omega$. We will show that any weak solution, with negative “enough” initial energy blows up in finite time. This work generalizes the result of Kirane and Tatar [5] to the multidimensional setting and includes an earlier result by the author [9] (see comments below).

To establish our result, we impose the following

H1) $c, m\in L^\infty(\Omega)$, $b, D, R\in[L^\infty(\Omega)]^n$, and for $a, c, k>0$ the functions A, K $\in[L^\infty(\Omega)]^{n\times n}$ satisfy

$$\begin{gathered} c(x)\ge c, \quad m(x)\ge m_0\ge 0, \quad \forall x\in\Omega. \\ A(x)\xi\cdot\xi\ge a|\xi|^2, \quad K(x)\xi\cdot\xi\ge k|\xi|^2, \quad \forall x\in\Omega,\ \forall\xi\in\mathbb{R}^n. \end{gathered}$$

H2) $(u_0, u_1, \theta_0)\in H_0^1(\Omega)\times L^2(\Omega)\times H_0^1(\Omega)$

H3) $p\le 2(n-1)/(n-2)$ if $n\ge 3$.

Definition By a weak solution of (1.1), we mean a pair (u,θ) such that

$$\begin{gathered} u\in C\left([0,T);H_0^1(\Omega)\right)\cap C^1\left([0,T);L^2(\Omega)\right) \\ \theta\in L^2\left([0,T);H_0^1(\Omega)\right)\cap C^1\left([0,T);L^2(\Omega)\right) \end{gathered} \tag{1.2}$$

and satisfying the system in the following sense [7]: For any $(v,\varphi)\in[H_0^1(\Omega)]^2$,

$$\begin{aligned} \frac{\partial}{\partial t}\int_\Omega u_t v\,dx &= \int_\Omega A(x)\nabla u\cdot\nabla v\,dx + \int_\Omega vb(x)\cdot\nabla\theta\,dx \\ &\quad + \int_\Omega vD(x)\cdot\nabla u\,dx - \int_\Omega vm(x)u_t\,dx + \int_\Omega ve^{\beta t}u|u|^{p-2}\,dx \\ \frac{\partial}{\partial t}\int_\Omega c(x)\theta\varphi\,dx &= \int_\Omega K(x)\nabla\theta\cdot\nabla\varphi\,dx + \frac{\partial}{\partial t}\int_\Omega ub(x)\cdot\nabla\varphi\,dx\int_\Omega\varphi R(x)\cdot\nabla u\,dx \end{aligned} \tag{1.3}$$

for almost every $t\in[0,T)$.

Remark. The condition on p in (H3) is imposed so that $\int_\Omega ve^{\beta t}u|u|^{p-2}\,dx$ makes sense.

2 Main Result

In this section we prove our main result. For this purpose we set

$$E(t) := \frac{1}{2}\int_\Omega [u_t^2 + A(x)\nabla u\cdot\nabla u + c(x)\theta^2]\,dx - \frac{1}{p}\int_\Omega e^{\beta t}|u|^p\,dx\,. \tag{2.1}$$

Lemma 2.1 *If* $E(0) < 0$ *and*

$$\beta \geq 2\sqrt{\frac{n(cd^2+r^2)}{4c}} \tag{2.2}$$

Then

$$E'(t) \leq -\int_\Omega K(x)\nabla\theta\cdot\nabla\theta\,dx \leq -k\int_\Omega |\nabla\theta|^2\,dx \leq 0. \tag{2.3}$$

Proof. By taking a derivative of (2.1) and using the equations of (1.1) we get

$$\begin{aligned} E'(t) &= \int_\Omega [u_t u_{tt} + A(x)\nabla u\cdot\nabla u_t + c(x)\theta\theta_t]\,dx \\ &\quad -\frac{\beta}{p}\int_\Omega e^{\beta t}|u|^p\,dx - \int_\Omega e^{\beta t}|u|^{p-2}uu_t\,dx \\ &= \int_\Omega u_t D(x)\cdot\nabla u - \int_\Omega m(x)u_t^2\,dx + \int_\Omega \theta R(x)\cdot\nabla u(x,t) \\ &\quad -\int_\Omega K(x)\nabla\theta\cdot\nabla\theta\,dx - \frac{\beta}{p}\int_\Omega e^{\beta t}|u|^p\,dx \end{aligned} \tag{2.4}$$

We then use Young's inequality and (H1) to obtain

$$\begin{aligned} E'(t) &\leq -k\int_\Omega |\nabla\theta|^2\,dx - m_0\int_\Omega u_t^2\,dx + \varepsilon_1\int_\Omega u_t^2\,dx + \frac{nd^2}{4\varepsilon_1}\int_\Omega |\nabla u|^2\,dx \\ &\quad +\varepsilon_2\int_\Omega \theta^2\,dx + \frac{nr^2}{4\varepsilon_2}\int_\Omega |\nabla u|^2\,dx - \frac{\beta}{p}\int_\Omega e^{\beta t}|u|^p\,dx \end{aligned} \tag{2.5}$$

where $d := \|D\|_\infty$ and $r := \|R\|_\infty$. By using (2.1) we obtain

$$\begin{aligned} E'(t) &\leq -k\int_\Omega |\nabla\theta|^2\,dx + 2(\varepsilon_1 - m_0)E(t) + [\varepsilon_2 - c(\varepsilon_1 - m_0)]\int_\Omega \theta^2\,dx \\ &\quad -[a(\varepsilon_1 - m_0) - \frac{n}{4}(\frac{d^2}{\varepsilon_1} + \frac{r^2}{\varepsilon_2})]\int_\Omega |\nabla u|^2\,dx \\ &\quad -\frac{1}{p}[\beta - 2(\varepsilon_1 - m_0)]\int_\Omega e^{\beta t}|u|^p\,dx \end{aligned} \tag{2.6}$$

At this point we take $\varepsilon_2 = c(\varepsilon_1 - m_0)$ and ε_1 large enough so that

$$a(\varepsilon_1 - m_0) - \frac{n}{4}(\frac{d^2}{\varepsilon_1} + \frac{r^2}{c(\varepsilon_1 - m_0)}) \geq 0.$$

It suffices in this case to have

$$a(\varepsilon_1 - m_0) - \frac{n}{4}\frac{cd^2 + r^2}{c(\varepsilon_1 - m_9)} \geq 0, \tag{2.7}$$

which is equivalent to

$$\varepsilon_1 \geq m_0 + \sqrt{\frac{n(cd^2 + r^2)}{4c}} \tag{2.8}$$

By combining all above and using (2.2) we arrive at

$$E'(t) \leq -k \int_\Omega |\nabla\theta|^2 \, dx + 2(\varepsilon_1 - m_0)E(t).$$

Therefore (2.3) is established provided that $E(t) \leq 0$. This is of course true since $E(0) \leq 0$.

Lemma 2.2 *Suppose that (H3) holds. Then there exists a positive constant $C > 1$ depending on n, p only such that*

$$\|u\|_p^s \leq C \left(\|\nabla u\|_2^2 + \|u\|_p^p\right) \tag{2.9}$$

for any $u \in H_0^1(\Omega)$ and $2 \leq s \leq p$.

Proof. If $\|u\|_p \leq 1$ then $\|u\|_p^s \leq \|u\|_p^2 \leq C\|\nabla u\|_2^2$ by Sobolev embedding theorems. If $\|u\|_p > 1$ then $\|u\|_p^s \leq C\|u\|_p^p$. Therefore (2.9) follows.

As a result of (2.1), (2.9), and the lemma, we have

Corollary 2.3 *Assume that (H3) holds. Then we have*

$$\|u\|_p^s \leq C \left(E(t) + \|u_t\|_2^2 + e^{\beta t}\|u\|_p^p + \|\theta\|_2^2\right) \tag{2.10}$$

for any $u \in H_0^1(\Omega)$ and $2 \leq s \leq p$.

Theorem 2.4 *Let (H1) and (H3) be fulfilled. Then given $T > 0$ there exists $\lambda > 0$ such that, for any initial data satisfying (H2) and*

$$E(0) < -\lambda, \tag{2.11}$$

the solution (1.2) blows up in a time $T^* \leq T$.

Proof. Set $H(t) = -E(t)$. Then, by virtue of (2.3), $H'(t) \geq k \int_\Omega |\nabla\theta|^2\,dx \geq 0$; hence

$$\lambda < -E(0) = H(0) \leq H(t) \leq \frac{b}{p} e^{\beta t} \|u\|_p^p. \tag{2.12}$$

We then define

$$L(t) := H^{1-\alpha}(t) + \varepsilon \int_\Omega u u_t(x,t)\,dx + \frac{\varepsilon}{2} \int_\Omega m(x) u^2(x,t)\,dx \tag{2.13}$$

for ε small to be chosen later and $\alpha = (p-2)/2p$. By taking a derivative of (2.13) and using equation (1.1) we obtain

$$\begin{aligned} L'(t) &= (1-\alpha)H^{-\alpha}(t)H'(t) - \varepsilon \int_\Omega A(x)\nabla u \cdot \nabla u\,dx \\ &\quad + \varepsilon \int_\Omega u_t^2 + \varepsilon e^{\beta t} \int_\Omega |u|^p\,dx + \varepsilon \int_\Omega u b(x) \cdot \nabla\theta\,dx \qquad (2.14) \\ &= (1-\alpha)H^{-\alpha}(t)H'(t) + \varepsilon \int_\Omega u_t^2 - \varepsilon \int_\Omega A(x)\nabla u \cdot \nabla u\,dx \\ &\quad + \varepsilon \int_\Omega u b \cdot \nabla\theta\,dx + \varepsilon p \left[H(t) + \frac{1}{2} \int_\Omega [u_t^2 + A(x)\nabla u \cdot \nabla u + c(x)\theta^2]\,dx \right] \end{aligned}$$

Then use Young's inequality to estimate $\int_\Omega u b \cdot \nabla\theta\,dx$ in (2.14). For all $\delta > 0$,

$$\begin{aligned} L'(t) &\geq k(1-\alpha)H^{-\alpha}(t)\|\nabla\theta\|_2^2 + \varepsilon(\frac{p}{2} - 1) \int_\Omega A(x)\nabla u \cdot \nabla u\,dx \\ &\quad + \varepsilon(\frac{p}{2} + 1) \int_\Omega u_t^2\,dx + \frac{p\varepsilon}{2} \int_\Omega c(x)\theta^2\,dx + p\varepsilon H(t) \\ &\quad - B\varepsilon \left[\frac{1}{4\delta} \|\nabla\theta\|_2^2 + \delta \int_\Omega u^2\,dx \right] \qquad (2.15) \\ &\geq \left[k(1-\alpha)H^{-\alpha}(t) - \frac{B\varepsilon}{4\delta} \right] \|\nabla\theta\|_2^2 + \varepsilon(\frac{p}{2} - 1) \int_\Omega A(x)\nabla u \cdot \nabla u\,dx \\ &\quad + \varepsilon(\frac{p}{2} + 1) \int_\Omega u_t^2 + p\varepsilon H(t) + \frac{p\varepsilon}{2} \int_\Omega c(x)\theta^2\,dx - B\varepsilon\delta \|u\|_2^2, \end{aligned}$$

where $B = \|b\|_\infty$. We then take $\delta = H^\alpha(t)/M$, for large M to be specified later. Substitute in (2.15) to arrive at

$$\begin{aligned} &L'(t) \qquad (2.16) \\ &\geq \left[k(1-\alpha) - \frac{M}{4}\varepsilon B \right] H^{-\alpha}(t)\|\nabla\theta\|_2^2 + \varepsilon(\frac{p}{2} - 1) \int_\Omega A(x)\nabla u \cdot \nabla u\,dx \\ &\quad + \varepsilon(\frac{p}{2} + 1) \int_\Omega u_t^2 + \frac{p\varepsilon}{2} \int_\Omega c(x)\theta^2\,dx + \varepsilon \left[pH(t) - \frac{B}{M} H^\alpha(t)\|u\|_2^2 \right]. \end{aligned}$$

By (2.10) and the inequality $\|u\|_2^2 \leq C\|u\|_p^2$, we obtain

$$H^\alpha(t)\|u\|_2^2 \leq C \left(\frac{b}{p} \right)^\alpha e^{\alpha\beta t} \|u\|_p^{2+\alpha p} \leq C_T \|u\|_p^{2+\alpha p};$$

where $C_T = C\left(\frac{b}{p}\right)^{\alpha} e^{\alpha\beta T}$; consequently (2.16) yields

$$\begin{aligned} L'(t) & \qquad\qquad (2.17) \\ \geq & \left[k(1-\alpha) - \frac{M}{4}\varepsilon B\right] H^{-\alpha}(t)\|\nabla\theta\|_2^2 + \varepsilon(\frac{p}{2}-1)\int_\Omega A(x)\nabla u\cdot\nabla u\,dx \\ & +\varepsilon(\frac{p}{2}+1)\int_\Omega \rho(x)u_t^2 + \frac{p\varepsilon}{2}\int_\Omega c(x)\theta^2\,dx + \varepsilon\left[pH(t) - \frac{B}{M}C_T\|u\|_p^{2+\alpha p}\right]. \end{aligned}$$

We then use corollary 2.3, for $s = 2+\alpha p < p$, to deduce from (2.17) that

$$\begin{aligned} L'(t) \geq & \left[k(1-\alpha) - \frac{M}{4}\varepsilon B\right] H^{-\alpha}(t)\|\nabla\theta\|_2^2 + \varepsilon(\frac{p}{2}-1)\int_\Omega A(x)\nabla u\cdot\nabla u\,dx \\ & +\varepsilon(\frac{p}{2}+1)\int_\Omega u_t^2 + \frac{p\varepsilon}{2}\int_\Omega c(x)\theta^2\,dx \qquad (2.18) \\ & +\varepsilon\left[pH(t) - \frac{B}{M}C_T\left\{H(t) + \|u_t\|_2^2 + \|u\|_p^p + \|\theta\|_2^2\right\}\right]. \end{aligned}$$

At this point, we choose M large enough so that (2.18) becomes

$$L'(t) \geq \left[k(1-\alpha) - \frac{M}{4}\varepsilon B\right] H^{-\alpha}(t)\|\nabla\theta\|_2^2 + \varepsilon\Gamma_1\left[H(t) + \|u_t\|_2^2 + \|u\|_p^p + \|\theta\|_2^2\right], \tag{2.19}$$

where $\Gamma_1 > 0$ is a constant depending on C_T (hence on T). Once M is chosen we then pick ε small enough so that $k(1-\alpha) - \varepsilon BM/4 \geq 0$ and

$$L(0) = H^{1-\alpha}(0) + \varepsilon\int_\Omega u_0u_1(x)\,dx + \frac{\varepsilon}{2}\int_\Omega m(x)u_0^2(x)\,dx > \frac{\lambda}{2}; \tag{2.20}$$

therefore (2.19) takes the form

$$L'(t) \geq \Gamma\left[H(t) + \|u_t\|_2^2 + \|u\|_p^p + \|\theta\|_2^2\right], \tag{2.21}$$

where $\Gamma = \Gamma_1\varepsilon > 0$; hence $L(t) \geq L(0) > \frac{\lambda}{2}$ for all $t \geq 0$. Now the estimate

$$|\int_\Omega uu_t\,dx| \leq \|u\|_2\|u_t\|_2 \leq C\|u\|_p\|u_t\|_2; \tag{2.22}$$

implies

$$\Big|\int_\Omega uu_t\,dx\Big|^{1/(1-\alpha)} \leq C\|u\|_p^{1/(1-\alpha)}\|u_t\|_2^{1/(1-\alpha)}.$$

Again Young's inequality, by virtue of corollary 2.3, yields

$$\begin{aligned} \Big|\int_\Omega uu_t\,dx\Big|^{1/(1-\alpha)} & \leq C\left[\|u\|_p^p + \|u_t\|_2^2\right] \qquad (2.23) \\ & \leq C_T\left[H(t) + \|u\|_p^p + \|u_t\|_2^2 + \|\theta\|_2^2\right]. \end{aligned}$$

Now we have the estimate

$$|\int_\Omega m(x)u^2\,dx|^{1/(1-\alpha)} \le C\|u\|_p^{2/(1-\alpha)} \le C_T\left[H(t)+\|u\|_p^p+\|u_t\|_2^2+\|\theta\|_2^2\right], \tag{2.24}$$

since $2/(1-\alpha) < p$. Finally by noting that

$$L^{1/(1-\alpha)}(t) \le 2^{1/(1-\alpha)}\Big(H(t)+|\int_\Omega uu_t\,dx|^{1/(1-\alpha)}|\int_\Omega m(x)u^2\,dx|^{1/(1-\alpha)}\Big)$$

and combining it with (2.21), (2.23) and (2.24) we arrive at

$$L'(t) \ge \gamma L^{1/(1-\alpha)}(t), \forall t \ge 0, \tag{2.25}$$

where γ is a positive constant depending on all above constants and Ω. A simple integration of (2.25) over $(0,\, t)$ then yields

$$L^{(p-2)/(p+2)}(t) \ge \frac{1}{L^{(p-2)/(p+2)}(0)-\gamma t(p-2)/(p+2)}$$

Therefore by choosing λ, the constant in (2.11), large enough $L(t)$ blows up in a time $T^* \le T$.

Remarks

1. If $d = r \equiv 0$, the system (1.1) reduces to the one in [9] and the blow up occurs for any initial data satisfying $E(0) < 0$ even if $m \ne 0$ and $\beta = 0$. Such a result cannot be obtained with the method used in [5].
2. We do not require that u_0 and u_1 to be cooperative as in (17) of [5] but instead we take initial conditions with negative 'enough' initial energy. Such initial data can be easily constructed using a lemma by Levine and Sacks [6].
3. It appears from the calculations above and the ones in [5] that, in the general case, a forcing term of the form $|u|^{p-2}u$ is not enough to make the blow up occurs unless some extra conditions on the initial data are added (see paragraph 5 of [5]).
4. Note that condition (13) of [5], namely $uf(t,u) \ge (2+4\gamma)F(t,u)$ for $\gamma \ge \gamma_0$, is omitted. Our forcing term only satisfies $uf(t,u) = pF(t,u)$, for any p however close to 2. In fact looking carefully to the calculations we easily see that condition (13) of [5] is dictated by the method itself (see also [10], [11]).
5. The above blow up result remains valid for more general forcing term $F(t,u)$ instead of $e^{\beta t}|u|^{p-2}u$. The choice of the latter one is only for simplicity.
6. Our method seems to be natural for systems with variable coefficients. In fact if m is not constant (even if $d = r = 0$) the use of the theorem by Kalantarov and Ladyzhenskaya [4] to establish the blow up seems to be difficult. One needs, at least, to reformulate condition (17) of [5]

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