$$\begin{vmatrix} & & 0 \\ 1 & -2-\lambda & -8 \\ 0 & -5 & 1-\lambda \end{vmatrix} = (3-\lambda)\begin{vmatrix} -2-\lambda & -8 \\ -5 & 1-\lambda \end{vmatrix}$$

$$= (3-\lambda)\left[(-2-\lambda)(1-\lambda) - 40\right]$$ (we have to multi[p]... out)

$$= (3-\lambda)\left[\lambda^2+\lambda-42\right] = (3-\lambda)(\lambda+7)(\lambda-6) = 0$$

$\Rightarrow \lambda = +6, 3, -7$ are the eigenvalues.

Eigenvectors

$\boxed{\lambda=+6}$ $\left(\begin{array}{ccc|c} -3 & 0 & 0 & 0 \\ 1 & -8 & -8 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right) \xrightarrow{R_1/3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & -8 & -8 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right] \xrightarrow{R_2-R_1}$

$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -8 & -8 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right) \xrightarrow[-R_3/5]{-R_2/8} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \xrightarrow{R_3-R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 1 & \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\Rightarrow x_1 = 0$ and $x_2 = -x_3$.

$\Rightarrow X = \begin{pmatrix} 0 \\ -x_3 \\ x_3 \end{pmatrix} = x_3\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \Rightarrow E_1 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ is an

eigenvector corresponding to $\lambda = 6$

$\boxed{\lambda=-7}$ $\left(\begin{array}{ccc|c} 10 & 0 & 0 & 0 \\ 1 & 5 & -8 & 0 \\ 0 & -5 & 8 & 0 \end{array}\right) \xrightarrow{R_1/10} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 5 & -8 & 0 \\ 0 & -5 & 8 & \end{array}\right) \xrightarrow[-R_3]{R_2-R_1}$

$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 5 & -8 & 0 \\ 0 & 5 & -8 & 0 \end{array}\right) \xrightarrow[R_2/5]{R_3-R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -8/5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\Rightarrow x_1 = 0, \quad x_2 - \frac{8}{5}x_3 = 0 \Rightarrow x_2 = \frac{8}{5}x_3$

$\Rightarrow X = \begin{pmatrix} 0 \\ \frac{8}{5}x_3 \\ x_3 \end{pmatrix} = x_3\begin{pmatrix} 0 \\ 8/5 \\ 1 \end{pmatrix} \Rightarrow E_2 = \begin{pmatrix} 0 \\ 8 \\ 5 \end{pmatrix}$

is an eigenvector.

Do the same for $\lambda = 3$.

problem #17

$$\begin{vmatrix} \alpha-\lambda & \beta \\ \beta & \gamma-\lambda \end{vmatrix} = \lambda^2-(\alpha+\gamma)\lambda+\alpha\gamma-\beta^2=0$$

The values of λ will be real iff the descriminant $\Delta \geq 0$

$$\Delta = (\alpha+\gamma)^2-4(\alpha\gamma-\beta^2)$$
$$= \alpha^2+\gamma^2-2\alpha\gamma+4\beta^2 = (\alpha-\gamma)^2+4\beta^2 \geq 0.$$

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problem #19 If λ is an eigenvalue of A, then there exists a vector $E \neq 0$ such that $AE = \lambda E$

$\Rightarrow A^2E = A(\lambda E) = \lambda AE = \lambda(\lambda E) = \lambda^2 E$

$\Rightarrow A^3E = A(\lambda^2 E) = \lambda^2 AE = \lambda^2(\lambda E) = \lambda^3 E$

we continue to get $A^k E = \lambda^k E$.

So λ^k is an eigenvalue of A^k, with E, a corresponding vector.

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problem #20 $AE = \lambda E$ and $AL = \mu L$, $E \neq 0$, $L \neq$

let $\alpha E + \beta L = 0$. So $A(\alpha E+\beta L) = 0 \Rightarrow$

$\alpha AE + \beta AL = 0 \Rightarrow \alpha\lambda E + \beta\mu L = 0$ (*)

If $\lambda \neq \mu$ then one of them is not zero (say λ)

Hence (*) $\Rightarrow \alpha E + \frac{\beta\mu}{\lambda} L = 0 \Rightarrow \alpha E = -\frac{\beta}{\lambda}\mu L$

replace in $\alpha E+\beta L = 0 \Rightarrow \beta\left(1-\frac{\mu}{\lambda}\right)L = 0 \Rightarrow$

$\beta = 0$ since $1-\frac{\mu}{\lambda} \neq 0$ and $L \neq 0$

using (*) again $\Rightarrow \alpha\lambda E = 0 \Rightarrow \alpha = 0$ since $\lambda \neq 0$ and $E = 0$ $\Rightarrow E, L$ are linearly independent.