

Section 7.7

Problem #9 Solve

$$14x_3 - 3x_5 + x_7 = 2$$

$$x_1 + x_2 + x_3 - x_4 + x_6 = -4$$

Sol. 
$$\left[ \begin{array}{cccccc|c} 0 & 0 & 14 & 0 & -3 & 0 & 1 & 2 \\ 1 & 1 & 1 & -1 & 0 & 1 & 0 & -4 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2}$$

$$\left[ \begin{array}{cccccc|c} 1 & 1 & 1 & -1 & 0 & 1 & 0 & -4 \\ 0 & 0 & 14 & 0 & -3 & 0 & 1 & 2 \end{array} \right] \xrightarrow{14R_1 - R_2}$$

$$\left[ \begin{array}{cccccc|c} 14 & 14 & 0 & -14 & -3 & 14 & 0 & -58 \\ 0 & 0 & 14 & 0 & -3 & 0 & 1 & 2 \end{array} \right] \xrightarrow{\begin{array}{l} R_1/14 \\ R_2/14 \end{array}}$$

$$\left[ \begin{array}{cccccc|c} 1 & 1 & 0 & -1 & -\frac{3}{14} & 1 & 0 & -\frac{58}{14} \\ 0 & 0 & 1 & 0 & -\frac{3}{14} & 0 & \frac{1}{14} & \frac{2}{14} \end{array} \right] \text{ so}$$

$$x_1 + x_2 - x_4 - \frac{3}{14}x_5 + x_6 = -\frac{58}{14} = -\frac{29}{7}$$

$$x_3 - \frac{3}{14}x_5 + \frac{1}{14}x_7 = \frac{1}{7}$$

The solution is

$$X = \begin{bmatrix} -\frac{29}{7} - x_2 + x_4 + \frac{3}{14}x_5 - x_6 \\ x_2 \\ \frac{1}{7} + \frac{3}{14}x_5 - \frac{1}{14}x_7 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -\frac{29}{7} \\ 0 \\ \frac{1}{7} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$+ x_4 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ \frac{3}{14} \\ \frac{3}{14} \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_7 \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{14} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

This is the general solution.

$$\left[ \begin{array}{cccc|c} -6 & 2 & -1 & 1 & 0 \\ 1 & 4 & 0 & -1 & -5 \\ 1 & 1 & 1 & -7 & 0 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[ \begin{array}{cccc|c} 1 & 4 & 0 & -1 & -5 \\ -6 & 2 & -1 & 1 & 0 \\ 1 & 1 & 1 & -7 & 0 \end{array} \right]$$

$$\begin{array}{l} R_2 + 6R_1 \\ R_3 - R_1 \end{array} \rightarrow \left[ \begin{array}{cccc|c} 1 & 4 & 0 & -1 & -5 \\ 0 & 26 & -1 & -5 & -30 \\ 0 & -3 & 1 & -6 & 5 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[ \begin{array}{cccc|c} 1 & 4 & 0 & -1 & -5 \\ 0 & \boxed{1} & -\frac{1}{3} & 2 & -\frac{5}{3} \\ 0 & 26 & -1 & -5 & -30 \end{array} \right]$$

$$\begin{array}{l} R_1 - 4R_2 \\ R_3 - 26R_2 \end{array} \rightarrow \left[ \begin{array}{cccc|c} 1 & 0 & \frac{4}{3} & -9 & \frac{5}{3} \\ 0 & 1 & -\frac{1}{3} & 2 & -\frac{5}{3} \\ 0 & 0 & \frac{23}{3} & -57 & \frac{40}{3} \end{array} \right] \xrightarrow{\frac{3R_3}{23}} \left[ \begin{array}{cccc|c} 1 & 0 & \frac{4}{3} & -9 & \frac{5}{3} \\ 0 & 1 & -\frac{1}{3} & 2 & -\frac{5}{3} \\ 0 & 0 & 1 & -\frac{171}{23} & \frac{40}{23} \end{array} \right]$$

$$\begin{array}{l} R_1 - \frac{4}{3}R_3 \\ R_2 + \frac{1}{3}R_3 \end{array} \rightarrow \left[ \begin{array}{cccc|c} 1 & 0 & 0 & \frac{21}{23} & -\frac{15}{23} \\ 0 & 1 & 0 & -\frac{17}{23} & \frac{155}{69} \\ 0 & 0 & 1 & -\frac{171}{23} & \frac{40}{23} \end{array} \right]$$

The solution: reduced system is

$$x_1 = -\frac{15}{23} - \frac{21}{23}x_4$$

$$x_2 = \frac{155}{69} + \frac{11}{23}x_4$$

$$x_3 = \frac{40}{23} + \frac{171}{23}x_4$$

The solution is

$$X = \begin{bmatrix} -\frac{15}{23} \\ \frac{155}{69} \\ \frac{40}{23} \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -\frac{21}{23} \\ \frac{11}{23} \\ \frac{171}{23} \\ 1 \end{bmatrix}, \quad x_4 \in \mathbb{R}.$$