Section 7.5

problem #3

$$\left(\begin{array}{ccc|c} -2 & 1 & 2 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ -2 & 1 & 2 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right) \xrightarrow[R_3 - R_1]{R_2 + 2R_1} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right)$$

$$\xrightarrow[\substack{-R_2 \\ R_3 + 2R_2}]{R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right) \xrightarrow{R_3/4} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\xrightarrow[R_2 + 2R_3]{R_1 + 2R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow x_1 = 0, \; x_2 = 0, \; x_3 = 0$$

So the only solution is the trivial solution

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problem #6

$$\left(\begin{array}{ccccc|c} 6 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & -2 & 0 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 2 & 0 \\ 6 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -2 & 0 \end{array}\right) \xrightarrow[R_3 - R_1]{R_2 - 6R_1}$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & -1 & 1 & -6 & -12 & 0 \\ 0 & 0 & 0 & \boxed{-1} & -4 & 0 \end{array}\right) \xrightarrow[R_2 - 6R_3]{\substack{-R_2 \\ R_1 + R_3}} \left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -2 & 0 \\ 0 & +1 & -1 & 0 & -12 & 0 \\ 0 & 0 & 0 & -1 & -4 & 0 \end{array}\right)$$

Reduced system

$$x_1 = 2x_5, \quad x_2 = x_3 + 12x_5, \quad x_4 = -4x_5$$

The solution is

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 2x_5 \\ x_3 + 12x_5 \\ x_3 \\ -4x_5 \\ x_5 \end{pmatrix} = x_3 \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 2 \\ 12 \\ 0 \\ -4 \\ 1 \end{pmatrix}$$

The solution space is

$$S = \left\{ \alpha \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 12 \\ 0 \\ -4 \\ 1 \end{pmatrix} ; \alpha, \beta \in \mathbb{R} \right\}$$

$\dim S = 2$ and $\operatorname{Rank} A = 3$