Section 6.5

problem 5

$u=(1,2,-3,1)$, $V=(4,0,0,2)$, $W=(6,4,-6,4)$

$\alpha u+\beta V+\gamma W = 0 \Rightarrow$

$(\alpha+4\beta+6\gamma,\ 2\alpha+4\gamma,\ -3\alpha-6\gamma,\ \alpha+2\beta+4\gamma)=(0,0,0,0) \Rightarrow$

$\alpha+4\beta+6\gamma=0$

$\left.\begin{array}{l}2\alpha+4\gamma=0\\ -3\alpha-6\gamma=0\end{array}\right\} \Longrightarrow \alpha+2\gamma=0 \Rightarrow \alpha=-2\gamma. \ (*$

$\alpha+2\beta+4\gamma=0$

Substitute in equations (1) and (4):

$\left.\begin{array}{l}4\beta+4\gamma=0\\ 2\beta+2\gamma=0\end{array}\right\} \Rightarrow \beta+\gamma=0 \Rightarrow \beta=-\gamma.\ (**)$

$(*)$ and $(**)$ $\Rightarrow$ the solution (the values) are (α,β,γ) such that $\alpha=-2\gamma$ and $\beta=-\gamma$.

If, for example $\gamma=1$, then $\alpha=-2$, $\beta=-1$

$\therefore$ u, V, W are linearly dependent.

problem # 14

$$\begin{vmatrix} 4 & 10 & 2\\ -3 & -3 & -6\\ 1 & 0 & 3\end{vmatrix} = +1\begin{vmatrix} 10 & 2\\ -3 & -6\end{vmatrix} - 0\begin{vmatrix} 4 & 2\\ -3 & -6\end{vmatrix} + 3\begin{vmatrix} 4 & 10\\ -3 & -3\end{vmatrix}$$

(+) (−) (+)

$$=(-60-(-6))+3(-12-(-30))$$

$$=-54+3(18)=-54+54=0$$

The vectors are linearly dependent.

problem 17.

$S = \{(x,y,-y,-x) \mid x,y \in \mathbb{R}\} \subset \mathbb{R}^4$.

It is easy to check that S is a subspace.

Any vector $u = (x,y,-y,-x) = x(1,0,0,-1) + y(0,1,-1,0)$

The vectors $\{1,0,0,-1), (0,1,-1,0)$ form a basis for S.

$\dim S = 2$.

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problem 24 S consists of all vectors // the plane $4x+2y-z$

$\Rightarrow$ $z = 4x+2y$.

So $u \in S$ has the form $u = (x,y,z) = (x,y,4x+2y)$

$= x(1,0,4) + y(0,1,2)$

It is easy to check that S is a subspace of $\mathbb{R}^3$.

$S = \{(x,y,4x+2y), \ x,y \in \mathbb{R}\}$.

A basis is $\{(1,0,4),(0,1,2)\}$

$\dim S = 2$.