problem # 15

To prove that $\|F+G\|^2+\|F-G\|^2=2\left(\|F\|^2+\|G\|^2\right)$ we compute:

$$\|F+G\|^2=(F+G)\cdot(F+G)$$
$$=\|F\|^2+2F\cdot G+\|G\|^2$$

Similarly

$$\|F-G\|^2=(F-G)\cdot(F-G)=\|F\|^2-2F\cdot G+\|G\|^2$$

By adding the equalities side to side, we get what we want.

problem #5

$(0,1,6,-4,1,2,9,-3)\cdot(6,6,-12,4,-3,-3,2,7)=$

$0+\cancel{6}-72-16-3-\cancel{6}+18-21=-94$

The angle between these 2 vectors is θ / $0\le\theta\le\pi$

and $\cos\theta=\dfrac{u\cdot v}{\|u\|\|v\|}=\dfrac{-94}{\sqrt{148}\,\sqrt{303}}$.

problem #8

$S=\{(x,2x,3x,y)\in\mathbb{R}^4 \mid x,y\in\mathbb{R}\}$

1°) If $x=y=0 \Rightarrow (0,0,0,0)\in S$

2°) $u=(a,2a,3a,b)$ and $v=(c,2c,3c,d)$

$u+v=\big(a+c,\,2(a+c),\,3(a+c),\,b+d\big)=(\alpha,2\alpha,3\alpha,\beta)\in S$

$\alpha=a+c,\ \beta=b+d.$

3°) $ku=(ka,2ka,3ka,kb)=(\gamma,2\gamma,3\gamma,\delta)\in S$

$\gamma=ka,\ \delta=kb\in\mathbb{R}$

So S is a subspace of $\mathbb{R}^4$.

In fact: any $u\in S$ has the form $u=(x,2x,3x,y)$

$=x(1,2,3,0)+y(0,0,0,1)$

It is clear that $e_1=(1,2,3,0)$ and $e_2=(0,0,0,1)$

are linearly independent becausse

$\alpha_1e_1+\alpha_2e_2=(\alpha_1,2\alpha_1,3\alpha_1,\alpha_2)=(0,0,0,0)\Longrightarrow$

$\alpha_1=\alpha_2=0$.

A basis for S is $\{e_1,e_2\}$. $\dim S=2$.