Research Statement

My earlier work (up to 1991) before promotion to associate professor was concerned with purely theoretical aspects of some approximation problems in the complex domain. Here, we discussed asymptotic behavior of certain sequences of rational interpolants related to a class of analytic functions on $|z| = \rho$ with $\rho > 1$.

From 1992, I was inclined towards **computational approximation** theory. With *Iqbal*, we considered simple rational functions $f(z) = z^m/(z-\rho)$ with $\rho > 1, m \geq 0$ and established that the asymptotic distribution of the zeros of their "Taylor sections" and "Lagrange interpolants at uniformly distributed nodes on the circle $|z| = \sigma$ is similar. This result was illustrated graphically in the FORTRAN environment JP[1]. We also conjectured that our result hold for all functions analytic in $|z| > \rho$.

I worked on a Birkhoff interpolation problem with *Dikshit* and *Sharma*. Here, we considered the regularity of a **Birkhoff interpolation** problem on some non-uniformly distributed roots of unity. Several problems of the types $(0,m)$-interpolation and $(0,1,2,\ldots,r-2, r)$-interpolation were considered in our joint work. Theorems proved in JP[8] extend some of the results due to Brueck, de Bruin, Sharma and W Chen.

I developed interest in approximation problems subject to interpolatory constraints in 1991. Initially, I posed and solved a constrained minimization problem **in complex domain**. Our main theorem in JP[3] extends some results on equiconvergence by Walsh, Rivlin and Cavaretta *et al.* Another type of problem I considered in this direction was related to interpolation problem mixed with l_2- minimization over the class of certain rational functions JP[4]. This type of problem was discussed by Sharma & Ziegler over the class of polynomials.

I shifted my attention to approximation of **real-valued** functions. My initial work in this direction with *Iqbal* appeared in JP[5] where we introduced "Constrained Orthogonal Polynomials" and observed that 3-term recurrence relation may be applied in their construction. To establish a convergence result, we considered a slight modification of Weirestrass Approximation Theorem.

I used the term "**Interpolating orthogonal polynomials** (*IOP*) or **Orthogonal zero interpolants**" for the "Constrained Orthogonal Polynomials" in my later work. The construction of these interpolants is based on the **Steiltjes procedure** almost in a similar manner as we notice in the case of classical orthogonal polynomials. I tested the computational procedure in the MATLAB environment on several functions in order to determine their L_2-

approximation subject to derivative constraints JP[14]. For this purpose, I modified some **algorithms** proposed by Walter Gautschi. The complicated part in this work was to establish an L_2- **convergence** result. For this, I worked out the **uniform convergence** in the relevant setup in JP[9] with an approach different from that adopted by Paszkowski. This result now helps to establish the desired L_2-convergence for similar type of problems.

The notion of (*IOP*) was applied to determine the numerical solution of **integral equations** and a class of **optimal control problems**. I took this work jointly with *Chaudhry*, *Qadir* in JP[6] and with *Sadek* in JP[7]. In both cases, I also carried out the computational work in FORTRAN programming.

Recently, I modified the **Gauss Quadrature Rule** by including one or both end points of the interval of integration as fixed nodes. I attempted this problem with two different approaches which appeared in JP[13] and CP[10]. These rules are capable of using maximum information about the differentiability of the integrand at the endpoints of the interval by using the notion of *Interpolating Orthogonal Polynomials*. I was able to establish some results related to convergence, degree of exactness and error analysis in the proposed modifications. The structure of these rules is different from that given in **Gauss-Radau and Gauss-Lobatto formulae**.

Keeping in view the difficulty of students on $(\varepsilon-\delta)$-**definition of limit**, *Yushau* and I redefined this concept in terms of local approximation of a function with constants (i.e., 0-degree polynomials). We also pointed out the importance of introducing appropriate technology at an earlier stage of teaching calculus courses.

The **further directions** in which I am planning to apply the notion of *IOP* are related to

- <u>Extension of Erdos-Turan result</u> JP[15]: Here, I have constructed a sequence of approximating polynomials which includes the nodes for interpolation lying outside the interval of its convergence. This result may prove useful in the time delay problems in optimal control theory.

- <u>Approximation by Interpolating Orthogonal Exponential Polynomials on a semi-infinite interval</u>: We are utilizing the approach of *IOP* to approximate functions of exponential order over an infinite interval and established L_2-convegence.

- <u>Optimal Control Problems</u>: In collaboration with Professor Sadek, I intend to apply some of my work in optimal control problems related to the heat equation over time interval $[0,\infty)$.

- <u>Solution of Boundary Value Problems via mixing fixed nodes with free nodes of Gaussian Type</u>: Here, we plan to modify certain numerical methods by fixing one or both end points of the partitioning intervals as fixed nodes and rely on a less number of free nodes in the process of approximating solution.

I have been involved in KFUPM **research projects** since 1993. My first project as a co-investigator was with *Zaman*. Here, we studied the diffraction of SH-type elastic waves propagating along the plane interface between two dissimilar elastic half spaces. The resulting mixed boundary value problem was reduced to the Weiner-Hopf equation. The solution of the diffraction problem was presented in a closed form JP[2].

One of the objectives in my academic pursuits is ***to share ideas*** *or any kind of academic achievement with my junior colleagues and motivate them to work as a part of my research team*. I have completed research proposals as a principal investigator with Samman and Yushau on "**Students' Learning Process in Pre-Calculus and Calculus Courses at KFUPM: Identifications of Problems and Possible Remedies**" and with Al-Attas on"**Interpolating Orthogonal Exponential Polynomials: Theoretical & Computational aspects**". These proposals carried some ideas which I gathered during the last 8 years but was unable to work out in my individual capacity due to time limitation. Currently, I am working on two projects, one dealing with numerical solution of **boundary value problems** and other on **"Improving Students Academic Performance"**.

During the period of my administrative assignment of reorganizing the Prep-Year Math Program, I developed my interest in the area of **Math Education**. There I concentrated on the matters that deal with the use of technology or innovative teaching methods. Along with Yushau, Wessels and Mji I made some studies about

- computer-aided learning in mathematics which appeared in JP[10]
- teaching pre-calculus in English to the students joining KFUPM from the schools with an Arabic medium of instruction JP[11].
- factors contributing to mathematics achievements CP[3].
- Predictors of success in computer aided learning of mathematics CP[6].

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