Research Profile

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I. Research Contribution as an Assistant Professor (up to 1991)

1. Research profile: *A brief description*

I initiated working on a specific research problem in March 1984 for my PhD degree program at the University of Alberta. After defending my thesis in December 1985, I returned to Pakistan to resume my job at BZU. In 1988, I availed an opportunity for a faculty position at KFUPM. Here, I found a wide arena for contribution to various aspects of academics.

From 1986 to 1990, I focused on the work accomplished in my PhD thesis in order to refine the material therein for the sake of journal publications. The publications JP [17]-[21] are the outcome of this effort from my doctoral thesis.

I found KFUPM a very fertile place to carry out research work. Initially, I pursued some ideas to activate myself in research which, indeed, was a part of my job requirement. Two of my publications [22]-[23] belong to my work which was carried out during the period 1989-1991.

2. Theme of Ph D work and relevant publications

My PhD work was based on the asymptotic behavior of certain sequences of rational interpolants (*related to functions f analytic in the region*: $|z| < \rho$ with $\rho > 1$) having uniformly distributed nodes and poles in the complex plane. In certain cases, these interpolants were directly related to l_2 or L_2 minimization problems. Our main objective was to investigate the region of convergence of two sequences respectively based on certain interpolants and Taylor sections of *f*, and then determine the optimal radius of the circle in which the difference of the two sequences (*DS*) converges uniformly and geometrically to zero. The resultant radius in all cases was found to be larger than the radius of the circle in which *f*, the relevant function, was analytic. The papers JP[18], [21] and CP[11] are related to this theme.

We also investigated the behavior of (*DS*) in the case when poles and nodes of the interpolants were slightly perturbed from their relevant circles. This work appeared in JP[17]

Some quantitative estimates and sharpness results were exclusively explored as a part of the Ph D thesis project which appeared in JP[19]

We concluded the thesis with some indirect results to answer the following question: if the (*DS*) related to a function *f* with unknown region of analyticity is uniformly bounded in a given region, what can we say about the region of analyticity of *f*? This work appeared in JP[20].

3. Post PhD work up to 1991

Besides preparing the write-ups for journal publications related to my PhD thesis, I looked into some other dimensions related to the theory of equiconvergence. In collaborative work with Prof. Sharma, we were able to extend Lou's results related to polynomial interpolants to the case of rational interpolants in the sense of Hermite JP[22]. During the same period, I worked on finding a solution of a min-max problem related to an analytic function over a specific class of rational functions which are referred to as next-to-interpolatory rational functions. Asymptotic behavior of the resultant solution sequence was compared with that of rational interpolants. This result extended a theorem due to Motzkin and Sharma and appeared as JP[23].

II. Research Contribution as Associate Professor (after 1991)

A. Exploring areas beyond Ph D work.

Although my mind was always inclined to explore ideas related to computational and applicable research, I could not manage adequate time for this purpose. Over a period of 9 years (1994-1997 and then 1999-2005), I was intensively involved in administrative assignments (committee work) of the department, college and university. During this period, I managed some time off and on to work on some ideas applicable to numerical solution of integral equations and optimal control theory. I shared some of my ideas with my colleagues and as a consequence, we came up with some joint publications.

I made research visits at my own expenses to Canada, USA in 1991, 1997, 1998 and short visits to Sharjah in 2002, 2005 and 2006. These visits helped me to explore new directions in my research work.

Recently, I have presented my work at some international and regional forums.

B. Combination of computational and theoretical work

From 1992 onward, I was inclined towards computational approximation theory. As a first step, I collaborated with one of my colleagues Dr. M. Iqbal who had good command on FORTRAN programming. We considered rational functions of the form $f(z) = z^m/(z-\rho)$ with $\rho > 1$ and established that the asymptotic distribution of the zeros of their Taylor sections and Lagrange interpolants at uniformly distributed nodes is similar. This particular question was raised to me by Prof. A. Edrie during his visit in 1985 at the University of Alberta. Dr Iqbal and I illustrated our result graphically as well which was based on FORTRAN Programming JP[1].

Dr. Iqbal and I also worked on a problem related to L_2 – approximation of real-valued functions subject to interpolatory constraints and established a convergence result JP[5] which required a modified form of Weirestrass Approximation Theorem. Computational aspects of this problem were also discussed in this paper. Our work was based on the notion of "**Interpolating orthogonal polynomials** (*IOP*) or **Orthogonal zero interpolants**" which has its roots in the work of J. L. Walsh.

After attending my seminar on (*IOP*), Professor I. Sadek, whose research interest is related to approximate solution of optimal control problems brought up the idea of collaborative work. The main benefit I got from the collaborative work was not only having awareness with the topic of "Optimal Control" but also getting into the depth of computational work based on FORTRAN programming. I feel pleasure to acknowledge the motivation and insight which I got from Professor Sadek during the period of our collaborative work JP[7]. This work is cited by several researchers.

I was also able to use the notion of (*IOP*) in the numerical solution of integral equations with two of my colleagues: Dr. M. Anwar Chaudhry who had some ideas on Integral equations from his PhD work, and Prof. A. Qadir who works in diversified areas of Mathematics. I took up the entire task of developing the algorithm related to the problem and also its implementation through FORTRAN programming. This work appeared in JP[6].

D. *IOP* and Walsh-type equiconvergence

During my visit at Kent State University, USA in 1991, I happened to discuss a notion of constrained least squares approximants for a certain type of complex functions f with Prof. A. S. Cavaretta. During the period 1992-1994, I worked on a minimization problem subject to interpolatory constraints with nodes on a certain set of roots of unity. The asymptotic behavior of the sequence resulting from these optimal polynomials was examined with that of Taylor sections of f JP[3].

E. Birkhoff interpolation

During my visit to Canada in 1998, I worked on a problem related to Birkhoff interpolation with Prof. H. Dikshit and Prof. A. Sharma. Here, we considered the regularity of a Birkhoff interpolation problem on some non-uniformly distributed roots of unity. We determined the range of values of a in the complex plane which makes the problem of lacunary interpolation on the zeros of $(z^n + 1)(z - a)$ regular. Several problems of types $(0,m)$-interpolation and $(0,1,2,\ldots,r - 2, r)$-

interpolation were considered in our joint paper JP[8]. Theorems proved in our work extend some of the results due to R Brueck, M. de Bruin, A. Sharma and W Chen. This work is cited in several papers.

F. Computational work based on Interpolating Orthogonal Polynomials
Before I discuss this part of my work, it may be appropriate to describe these polynomials the way I perceive:

"*A sequence of Interpolating Orthogonal Polynomials consists of Orthogonal Polynomials with respect to a given weight function over an interval [c,d] which interpolate to the zero function on a finite number of pre-assigned simple or multiple nodes*"[1].

The construction of these interpolants is based on the Steiltjes procedure almost in a similar manner as we notice in the case of classical orthogonal polynomials. However, the involvement of integrals of higher degree monomials in the computation of these polynomials makes the procedure unstable. This difficulty was tackled by the standard discretization of integrals due to Fejer as well as Chebyshev. The computational procedure was tested in the MATLAB environment on several functions of different characteristics to determine their L_2-approximation subject to derivative constraints JP[14]. For this purpose, I modified some algorithms proposed by Walter Gautschi which are available on the internet.

G. Work Related to Interpolating Orthogonal Polynomials
The notion of (*IOP*) provided me with some further directions in the area of computational approximation theory. These polynomials are found helpful in solving a certain class of minimization problems subject to interpolatory constraints. The basic technique used here modifies the underlying approximating space and thus converts the constrained problem to an unconstrained one.

In order to establish L_2-convergence in the constrained minimization problems, I worked out the uniform convergence in the relevant setup with an alternative method JP[9]. This result now serves as a basic tool to establish the desired L_2-convergence for similar type of problems.

Recently, I modified the Gauss Quadrature Rule by including one or both end points of the interval of integration as fixed nodes. I attempted this problem with two different approaches which appeared in JP[13] and CP[10]. These rules are capable of using maximum information about the differentiability of the integrand at the endpoints of the interval by using the notion of *Interpolating Orthogonal Polynomials*. I was able to establish some results related to

[1] *Orthogonal Zero Interpolant* may be considered as an appropriate substitute for the terminology *Interpolating Orthogonal Polynomials*.

convergence, degree of exactness and error analysis in the proposed modifications. The structure of these rules is different from that given in Gauss-Radau and Gauss-Lobatto formulae.

H. Future Work Related to Interpolating Orthogonal Polynomials

The other directions in which I am applying the notion of *IOP* are related to

- Extension of Erdos-Turan result JP[15]: Here, I have constructed a sequence of approximating polynomials which includes the nodes for interpolation lying outside the interval of its convergence. This result may prove useful in the time delay problems in optimal control theory.

- Approximation by Interpolating Orthogonal Exponential Polynomials on a semi-infinite interval: We are utilizing the approach of *IOP* to approximate functions of exponential order over an infinite interval and established L_2-convegence.

- Optimal Control Problems: In collaboration with Professor Sadek, I intend to apply some of my work in optimal control problems related to the heat equation over time interval $[0,\infty)$.

- Solution of Boundary Value Problems via mixing fixed nodes with free nodes of Gaussian Type: Here, we plan to modify certain numerical methods by fixing one or both end points of the partitioning intervals as fixed nodes and rely on a less number of free nodes in the process of approximating solution.

I. First KFUPM research project

I got involved in KFUPM approved research project first time in 1993 as a co-investigator. Prof. F. Zaman was the principal investigator of the project. Here, we studied the diffraction of SH-type elastic waves propagating along the plane interface between two dissimilar elastic half spaces. The resulting mixed boundary value problem was reduced to the Weiner-Hopf equation. As a consequence, the solution of the diffraction problem was presented in a closed form JP[2].

J. Funded Research Proposals

Since 2002, I submitted three funded research proposals in collaboration with colleagues who were fresh PhD graduates. Here, I would like to stress that *one of the objectives in my academic pursuits is to share my ideas or any kind of academic achievement with my junior colleagues and motivate them to work as a part of my research team*. I submitted research proposals as a principal investigator with M. Samman, H. Al-Attas and B. Yushau. These proposals carried some ideas which I gathered during the last 8 years but was unable to work out in my individual capacity due to time limitation. Currently, I am also a part of a team working on the KFUPM project "Improving Students Academic Performance"[..].

K. (L,ε)-approximation of limit: An *alternative way to define limit*

Since the time I have been teaching $(\varepsilon-\delta)$-definition of limit, many freshman students question its utilization while they have to concentrate only on the techniques that help them to solve limit problems. In fact, I find many freshman students to be frustrated when I present this definition to them in the calculus courses. It is, indeed, based on abstraction involving many parameters and has a complicated procedure to establish $\lim_{x\to a} f(x)=L$ even for very simple functions like $f(x)=x^2$. With the involvement of my colleague who happened to be my PhD student at that time, we redefined the concept of limit in terms of local approximation of a function with constants (i.e., 0-degree polynomials). We also pointed out the importance of introducing appropriate technology at an earlier stage of teaching calculus courses. In our work we demonstrated the use of "MAPLE" for certain type of limit problems where the underlying functions are either rational or transcendental. In this paper we showed that all the theorems concerning limits and the notion of continuity can be derived from our suggested approach.

We plan to extend this definition to functions of several variables.

L. Contribution in Math Education

During the period of my administrative assignment of reorganizing the Prep-Year Math Program, I developed my interest in the area of Math Education. I involved myself in the projects that deal with the use of technology or innovative teaching methods. With collaboration of my Ph D student who happened to be my colleague, I made some experiment-based studies about

- teaching pre-calculus in English to the students joining KFUPM from the schools with an Arabic medium of instruction JP[11].
- computer-aided learning in mathematics JP[10].
- factors contributing to mathematics achievements CP[3].
- Predictors of success in computer aided learning of mathematics CP[6].

III. Other Research Related Contributions

Besides my involvement in the KFUPM funded research projects and other individual or collaborative research activities, I participate and contribute in technical seminars under the auspices of Math Colloquium within and outside KFUPM. I also present my work at regional and international conferences.

I have been involved in MS/PhD thesis related activities as well. The details of all these activities are given in the CV.