King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

Students' Learning Process in Pre-Calculus and Calculus Courses at KFUPM

Identification of Problems and Possible Remedies

by

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Final Report

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Summary

This project deals with the issues related to Pre-Calculus and Calculus (**PCC**) courses at KFUPM. It was conducted in a form of experiment in the on-going Calculus classes during the academic terms 042 and 051. As narrated in the Project Proposal, one of its main objectives is to determine the students' deficiencies which they carry from the Pre-Calculus material at the time of entering Calculus I. Another objective of the project is more or less of similar nature, i.e., to identify students' retention level on basic Calculus concepts around the completion of a Calculus course. In order to achieve these objectives, we designed some surveys for the students taking Calculus I & II after completion of their Prep-Year Math courses. These surveys were based mostly on mono-concept questions which the students already covered in their past or current courses. Each survey was followed by a short test (which we shall refer as the Follow-up Quiz or Exit Quiz). The purpose of these tests was to authenticate the students' perception what they claimed to know or retain basic concepts introduced to them in the PCC courses. Based on the outcomes of these activities, we developed, as a part of our project's strategic plan, some remedial measures that, to certain extent, helped the students improving their level of understanding.

This report consists of nine chapters mainly dealing with the following topics:

- **A.** Global status of PCC courses with particular focus on their set-up at KFUPM
- **B.** Review of literature on Calculus reforms
- **C.** Implementation planning of the project
- **D.** Details of the experiment
- **E.** Statistical analysis of data based on surveys and tests
- **F.** Recommendations.

For further details on the nature of topics, please see the table of contents.

1. Introduction

1.1. Project and project team

During the year 2004, the Deanship of Academic Development (DAD) approved a project entitled “Students’ Learning Process in Pre-Calculus and Calculus Courses at KFUPM: Identification of Problems and Possible Remedies”. The team of three faculty members from the Department of Mathematical Sciences who proposed the project consists of: Dr. Muhammad Ashfaq Bokhari (Principal Investigator), Dr. Mohammad Samman (Co-Investigator) and Mr. Belarabie Yushau (Co-Investigator). This project, which has now been completed, deals with KFUPM students’ comprehension and command on the material related to Pre-Calculus and Calculus (PCC). It was conducted in the form of an experiment on the freshman students of KFUPM during the academic terms 042 and 051.

1.2. Calculus along with Pre-Calculus at undergraduate level

Universally, Calculus is regarded as the backbone of any BS program in the disciplines of Engineering, Computer and Physical Sciences. Not only that, its role is also quite significant in the social and management sciences. It would not be an exaggeration to state that the subject of Calculus initiates a solid process of learning among the students in different dimensions, forces them to utilize all faculties of their brain and above all, reshapes their mind to retain the material in this process. In the course of learning this subject, a student undergoes an extensive exercise of designing a logical stream of arguments required to solve a mathematical or physical problem. Calculus creates a geometric visualization of basic real life problems leading to their formulation as mathematical models. And finally, it helps the students in identifying analytic as well as numerical techniques helpful in finding the solutions of these models.

1.3. Literature Survey

The foundation of Calculus by and large relies on the Pre-Calculus material, which is usually distributed over two courses of College Algebra and Trigonometry. The deficiency of the students in the preparation of both Pre-Calculus and Calculus (**PCC**) has been registered over the past few decades in most of the developing as well as developed countries. An inadequate comprehension of the concepts and high failure rate among the students of PCC has been a matter of great concern for these countries till now. As a consequence, the US and most of the European countries launched several projects and held regular workshops that concentrate on the improvement of students' learning in the PCC courses [e.g. see National Council of Teachers of Mathematics (NCTM) 2000]. The theme of these projects and workshops, as we observe, vary from the development of conformable curriculum [e.g 1, 4, 7] to the improvement of teaching methodologies [e.g. 9, 15]; from the improvement of students' comprehension to the structure of meaningful evaluation system [e.g. 13]; from the use of technology [e.g. 13, 17] to the PCC applications that address the real life problems, and so on. It may be interesting to note that the concern shown towards the issues of PCC is not curtailed to projects and workshops. Comprehensive scientific research has also been conducted in several institutions of the US. As a result, we find quite a few Ph.D. theses [e.g. 2, 3, 6, 7, 8, 11] as an outcome of these efforts from reputed schools. In short, keeping in view the pivotal role of Calculus in almost all disciplines of university programs, its every aspect is being evaluated from different angles particularly in the US and the European countries. Particularly, Calculus Reforms has turned out an on-going process in the US. Various issues related to Calculus ranging from "planning and implementation of reformed curricula" to "use of technology" or from "comparison of learning methodologies" to "students' assessment" are presented at different forums [18 a-d] and appear in the literature [19 a-e] on a regular basis over there.

1.4. Status of PCC courses at KFUPM

It may be worthwhile to mention that the best high school graduates from the entire Kingdom of Saudi Arabia are admitted at KFUPM by observing a very stringent admission policy. These students at the time of entering into a BS program are immediately required to take two Pre-Calculus courses (Basic Algebra, Geometry, Functions, Trigonometry, and Introductory Linear Algebra) where English is the mandatory medium of instruction. The primary objective of these courses is to bridge a gap that exists between high school mathematics (which they mostly do in an Arabic medium of instruction) and the material prerequisite for the KFUPM Calculus Sequence Courses (Calculus I, II & III). The three Calculus courses are an integral part of BS curricula in the disciplines of engineering, computer and sciences.

1.5. Problems in PCC courses at KFUPM and improvement measures

Like other worldwide institutions, as indicated above, KFUPM also faces various kinds of problems related to the students' performance in PCC courses. Over the past few years there has been a consistent complaint from the Calculus instructors referring to the poor performance of students. This problem also persists beyond freshman level as instructors teaching Differential Equations (Math 202, Math 260) and Applied Mathematics (Math 301, Math 302) complain about the poor background of students on the PCC topics. This chain of complaints even extends up to senior level specialized courses that require an adequate PCC background from the students.

During the last decade, a higher failure rate and a significant increase in the D and D+ grades was noticed in Pre-Calculus courses which involve approximately 1500 new entrants to KFUPM every year. The university showed great concern to resolve this problem by setting several committees which were partly assigned the task of finding the cause(s) of this deterioration and suggesting some corrective measures. All of these committees came with some recommendations, some of which were implemented as well. Unfortunately however, no concrete improvement was noticed in this direction. As a last resort, the university administration, in pursuance of increasing the number of good quality students at the prep level, created an independent academic and administrative set-up for these courses which is known as "Prep-Year Math Program". Over the past few years, several innovative ideas were introduced in different directions with the hope of possible improvement in this set-up.

As far as the Calculus courses are concerned, the Department of Mathematical Sciences at KFUPM has been monitoring the students' academic accomplishments in these courses with extensive responsibility, particularly after the abolishment of the "integrated type of coordination" in these courses. Whenever the Math instructors observe a problem leading to poor performance of students in the PCC courses, they usually recommend corrective measures to overcome the problem. The Department tries its level best in paying attention to the issues raised by the faculty and finding the ways for implementation of their recommendations.

1.6. Identification of problems and pertinent corrective measures

The process of pinpointing a problem that is impairing a system plays a vital role in its flourishing. After that, an appropriate choice of corrective measure and its implementation needs due attention. These steps categorically lead to an improvement of a system. Otherwise, any measure without properly focusing on a problem would be a futile exercise and wastage of resources. The math faculty members at KFUPM, in general, do not appear very comfortable with the current performance of students in the PCC courses. On several occasions, during the academic terms as well as after the final exams, they have pointed out various kinds of student deficiencies that range from their high school preparation to study habits; language barrier to classroom attention; and time management to retention of concepts etc. Based on these observations, the Department usually constitutes ad-hoc committees that deal with issues like appropriate adjustment of course contents, suitable choice of PCC textbooks, mode of recitation classes for Calculus I & II, and the policies on students' assessment and evaluation. However, in spite of all the efforts, a noteworthy improvement has not yet been observed in the students' performance.

To the best of our observations, there are several points linked to both teacher and taught that are influencing the performance of the students in the PCC courses and require our due attention. These include measurement of students' comprehension level at the start of any PCC course, modification in teaching methodologies, interaction between PCC course-instructors during the term, and an evaluation process based on learning outcomes. With the above background about the global situation of PCC and having realized the situation of this problem at KFUPM in particular, we thought of the need to address the problem considering the following possible aspects:

1. Assessment of students’ comprehension level at the start of any PCC course.
2. The connection between the subject and the student.
3. Teaching methodology
4. Evaluation process.

1.7. Theme of the project

As explained in the proposal, the project team mainly addressed two issues related to students in the context of PCC courses, namely,

- Comprehension and retention of course material
- Identification of parameters affecting students’ performance in these courses.

The project was based on classroom experiments associated with the project and was carried out in two phases:

- ***Phase I***. During the Term 042 (*related to the students of MATH 102*)
- ***Phase II***. During the Term 051 (*related to the students of MATH 101*).

In both phases, we focused our attention on some specific teaching methodologies that may help students in retaining the concepts and techniques they learn in a PCC course. Also, we tried to design an evaluation process which hopefully measures the level of students’ attainment in a PCC course.

2. Literature Review Concerning Retention Level

A low level of students’ retention of definitions, concepts and theorems in a math course is a matter of great concern to relevant academicians. There are quite a number of studies that were conducted to investigate **various problems related to students’ retention**. The issue of retention of knowledge is an important area of research that has the potential to improve instructional practices and achieve school learning goals. Some studies have specifically addressed this aspect in the area of mathematics. One such study was conducted by Kwon, Oh Nam, Allen, Karen, Rasmussen, and Chris [12]. However, a few studies were carried out about Calculus at the university level.

Central questions regarding retention of mathematical knowledge were considered in [12]. Here, some discussion led to certain points that may help the educators to facilitate students' retention of mathematical understanding and skills for longer periods of time. Another type of question was directed to the relationship, if any, between instructional approaches and retention of mathematical knowledge.

Garner and Garner [5], compared outcomes of traditional and reformed calculus courses in terms of students' retention after a passage of time on basic concepts and skills. Among other findings, it was observed that students subject to reformed calculus courses retain better conceptual knowledge whereas the others (in traditional calculus) retain better procedural knowledge. In addition, the former (in reformed calculus) understand concepts before computational competence is achieved.

In [12], the authors investigated students' retention of mathematical knowledge and skills in two differential equations classes. Post-tests and delayed post-tests after 1 year were administered to students in inquiry-oriented and traditional classes. The results show that students in the inquiry-oriented class retained conceptual knowledge, as seen by their performance on modeling problems, and retained equal proficiency in procedural problems, when compared with students in the traditionally taught classes. These results add additional support to the claim that teaching for conceptual understanding can lead to longer retention of mathematical knowledge.

In another study, Jeanetta du Preez, Ansie Harding, Johann Engelbrecht [10] investigated the long-term effect of a technique mastering program in first year Calculus course which involved a group of first year engineering students at the University of Pretoria. This study investigated *which and how much of the knowledge and skills embedded by the technique mastering program in the first year, is retained after a further two years of study*. A quantitative and qualitative investigation shows that, in general, there is a disappointing decline in performance over a period of two years. There are, however, areas in which students performed better after the elapsed period. The research is of diagnostic value in determining the future of the technique mastering program with regard to its content.

As far as the retention of material issue is concerned, there are many aspects related to **the review of previous material** that would be considered. One aspect, for instance, is the reasons of reviewing.

There are many **reasons for the reviewing** to be considered. The review

- promotes continuity and helps students to attain a more comprehensive view of the mathematical topics covered.
- summarizes main ideas, develop generalizations, and get an overall view of what they have been learning piecemeal.
- helps students to assimilate or consolidate what they have learned, enabling them to fit ideas into new patterns.
- serves as a diagnostic tool, revealing weaknesses and strengths to students and teachers.
- helps teachers identify what is already known and what is not yet known; then re-teaching can be planned.
- assures that the prerequisites needed for learning new content have been mastered.
- adds to students' confidence in their ability to move successfully to new mathematical topics.

Again, when we discuss the reviewing issue, we may think about the **timing of review.** Most textbooks incorporate review sections at both the beginning and the end of most chapters. Some researchers take exception to this pattern, terming it "spastic." They argue that review should be continuous. Instead of presenting 25 problems on a new topic, it is suggested that only three or four be given, along with 25 review problems. Based on the outcome of their surveys, they claimed that this approach helps students attain a high level of understanding.

Research clearly indicates that review should be systematically planned and incorporated into the instructional program. Before a new topic or unit is begun, an inventory can help the teacher ascertain whether any prerequisite knowledge is missing. Such a review also helps students pull together the mathematical ideas they will need for the new topic. The inventory should range from simpler skills and concepts to the most difficult in order to pinpoint those which need to be re-taught. It should be noticed that a daily review of homework alone is not sufficient; it often concerns only a small portion of the needed prerequisites for a new topic.

After a topic or unit is taught, key points or objectives should be reviewed. Students thus become aware of the major highlights of the lesson so they can focus on the mathematical skills or concepts that will be needed in future lessons. It should be made clear to students that this is not simply a collection of exercises and problems; the review includes those topics which are the most important to remember.

Long-term retention is best achieved if assignments on a particular skill are spread out in time, rather than concentrated within a short interval. Reviewing immediately after instruction consolidates the ideas from that instruction, while delayed reviewing aids in the re-learning of forgotten material. Additionally, research has indicated that short periods of intensive review are better than long periods. Interspersing review throughout the textbook or curriculum is better than concentrating review at one time.

Once again, when we consider the reviewing issue, we have to think about the different possible **types of review.** One type of review is **outlining.** The process of outlining forces students to organize ideas and provides a structure that will help students put ideas together. When students use the outline they have made as an aid in reviewing, restructuring or recall of the mathematical ideas is promoted.

Review questions. Another type of review is solving relevant **questions.** Several researchers have focused on review questions, finding them effective for revealing content that has not been meaningful to students. Students who used the review questions scored significantly higher than those using textbook drill pages. Review questions were found to facilitate retention by promoting comprehension. Feedback helped students to consolidate what they had learned in the course of answering the review questions, and they developed their ability to transfer problem-solving skills. The word-type review questions required a thorough understanding of the concepts and rules and the ability to apply them to new situations. On the other hand, the calculation-type review questions required only comprehension of a narrow range of concepts, rules, and often focused on rote learning.

Testing. Another way of review is **testing**. A study sought to determine if the learning and retention of mathematical knowledge was affected by any of the three types of review procedures: testing, testing-with-explanation, and unit review. For the unit review, a list of behavioral objectives for the topics taught was given to students, with a worked example and an optional exercise identical to the corresponding test item. The test group was only given a test, and then the answers. Both the unit review and testing-with-explanation groups were given worked-out solutions for test items and optional exercises. The reviews enhanced the learning and retention of mathematics, with testing-with-explanation the most promising and testing alone the least promising. As in other studies, the usefulness of feedback in promoting achievement was apparent.

Homework. A traditional way of review is the **homework**. The role of differing types of homework designed for review in first-year algebra classes was explored by some researchers. In this type of research, one group was given homework consisting of exercises related to the topic taught that day. Another group was assigned fewer exercises related to the topic taught that day, plus exploratory exercises on content taught on each of the prior two days and review exercises on the first and third days after the teaching of a topic. The group having exploratory and review exercises achieved and retained better than the group having exercises related only to the daily topic.

It is worthwhile mentioning, that in our project, as will be observed in the latter chapters, our approach of addressing the PCC based problems and exploring subsequent remedial measures more or less conform to those appearing in the literature.

3. Implementation Plan

3.1. Responsibilities

In Phase I, the experiment was carried out in sections 3, 6, 14 and 19 of MATH 102 (Term 042). It may be noted that MATH 102 is a 4-credit hour course with three lectures/wk and one recitation class/wk[1]. The principal investigator (Dr. M.A. Bokhari) and the co-investigator (Dr. M. Samman) acted as the course-instructors respectively for the sections 3, 6 and 14, 19 whereas the second co-investigator (Mr. B. Yushau) was responsible to conduct the "Problem Solving Classes" for these sections.

Similar responsibilities were undertaken by the project team in Phase II of the experiment for four sections of MATH 101 during the term 051.

As pointed out in the Project Proposal, the course contents and the contact hours (in terms of number of classroom lectures/recitation classes) were maintained according to the existing policy of the Department during the period of the experiment. The project members maintained close interaction among themselves while implementing the plan.

3.2. Initial planning

The project team initiated the project in accordance with the plan given in the Revised Proposal which was submitted in May 2004 to DAD[2]. The project team held several meetings to accomplish the following tasks related to MATH 102 (Term 042) during the period October 2004-January 2005:

- Literature survey
- Preparation of experiment for Phase I.
- Mode of recitation classes
- Use of technology

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[1] This class is also referred to as a problem solving class in which students are supposed to go over the problems of the course material covered during the preceding week.

[2] Execution of this plan was further clarified in a memo addressed to the Dean (Academic Development) dated October 04, 2004.

3.2.a. *Literature Survey*

The team went over some reports on Calculus related matters available in the Department, some papers on Calculus Reforms which appeared in the proceedings of the "Orlando Sessions on Calculus Reforms (1996)"and some material on Calculus which appeared in the MAA[3] publications [1, 15-17, 19] and abstracts of some PhD dissertations related to Calculus [6-8, 11, 14]. In addition, the team looked into some material which specifically addressed the problem of students' retention (*See Chapter 2 for the details*)

3.2.b. *Preparation of Experiment for Phase I*

The project team analyzed the material of MATH 102 in order to identify the concepts from the prerequisite courses MATH 001/002/101, which are directly related to the contents of MATH 102. A survey was designed on the basis of these concepts (*See Appendix* 1(i)).

3.2.c. *Mode of Recitation Classes*

A plan for recitation classes was prepared to create momentous interaction among students and the recitation class instructor. A specific strategy was planned in order to utilize the recitation classes with respect to the following aspects

- Group Activity
- Use of Math software
- Maintaining close interaction between the course instructors and the recitation class instructor.

3.2.d. *Use of Technology*

A joint WebCT account was arranged for the allocated sections of MATH 102. The main components of the WebCT were designed for the enhancement of the project plan. It was decided to introduce the software "Mathematica" just for awareness in the recitation classes. Therefore, a handout summarizing the main commands of Mathematica was prepared to facilitate the students' learning (*See Appendix* 3(i)). Keeping in view the

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[3] Mathematical Association of America

length of the course, this activity was kept at a minimal level so that it does not affect the course-coverage.

4. Activities during Term 042 (Phase I)

4.1. Survey I & Related Activities

4.1a. *Survey I*

Survey I was conducted among the MATH 102 students during the 1st week of classes. It aimed to determine awareness of the students on the basic concepts of MATH 001/002/101 that are frequently used in MATH 102. The students were asked to tick the appropriate box given against each survey item in case they understand the concept. **416 students** from **18 sections** of MATH 102 participated in the survey. In Appendix 1(i), the figure given in a box against each question represents the percentage of students who claimed that they were aware of the concept.

4.1b. *Follow-up Quiz*

In order to authenticate the survey, a quiz was prepared from the survey items where at least 75% of the students claimed their awareness. This quiz was given to the sections of those students who participated in the survey. **329 students** sat in the quiz. It was conducted during the 1st week of the classes. The Follow-up Quiz and its outcome may be found in *Appendix* 1(ii).

4.1c. Preparation of Remedial Material

The concepts related to Survey I and the Follow-up Quiz where the students had a poor response were classified as a part of Remedial Material. This material was posted on the WebCT for students' self-study. It is reiterated that the deficiency material identified from the contents of MATH 001/002/101 was required in various parts of MATH 102 (*See Appendix* 3 (ii)).

4.2. Some Teaching Related Activities

4.2a. *Teacher-Student Based Learning*

The Project Team adopted the following format of teaching for the coverage of course material:

After introducing a new concept in the class, the relevant exercises were divided into three parts:

- Problems solved in the class lectures with partial involvement of students.
- A set of similar exercises for "Recitation Classes" involving group activity as well.
- Homework problems similar to text exercises but with changed parameters.

The students were encouraged to consult the course instructor in case of difficulty on homework or problems assigned for recitation classes.

A good number of students (not all) appreciated the structure of exercises. As a follow-up, the students were given a quiz that was based on the material they covered in the preceding week(s). The solution of quizzes and homework was posted on the WebCT. It was noted that the students considered some of the homework problems challenging for them. Those who put an effort in doing the homework came up with a good score on the quizzes. Overall, the students made a better score on quizzes as compared to that on homework.

4.2b. *Recitation Classes*

The students were divided into various groups in each class. Each group consisted of 5-6 students with lower to higher achievements in their earlier math courses. The weekly recitation classes involved the following activities:

- Group activity for problem solving (20 minutes)
- Presentation of solution on board by randomly selected students (15-20 minutes)
- Pop quiz out of recitation material and/or explanation by the instructor (10-15 minutes)
- Assigning 1-2 exercises based on use of "Mathematica" (outside the class)

Further details on "Recitation Classes" may be found in *Appendix* 3(iii)

4.2c. *Use of WebCT and Overhead Projector*

Based on past experience, the project team identified part of the course material which used to create a misunderstanding of concepts among students due to the overlapping of concept/terminology, e.g.,

- Volume of solid of revolution by different methods
- Various convergence tests for infinite series
- Different methods for power series representation of functions
- Solution of an integral by different techniques.

This type of material was presented by an overhead projector and students were given a chance to observe similarities and differences in closely related concepts and the relevant techniques while solving the exercises. Solutions of some exercises were left incomplete during the class lectures. It forced most of the students to practice on the partially solved problems outside the classroom. The students were able to cover a large variety of problems in the class by this approach. In some cases, part of the lecture material was posted on the WebCT for the sake of students' quick revision. This material was particularly related to the concepts where the students were found in confusion. The students were advised at the same time to go through the textbook and read the corresponding examples therein as well.

4.3. Survey II & Midterm Exam

4.3a. *Survey II*

Student Survey II was designed only for the students of the Project Team (i.e, Sec. 3, 6, 14 and 19). This survey was conducted during the 7th week of the term. The purpose of the survey was to determine the level of retention of the course material allocated for the mid term exam. **100 students** were present when the survey was conducted.

4.3b. *Students' awareness with their deficiency*

It may be noted that the students were not informed about the outcome of Survey II. However, some did realize their deficiency on some concepts while filling in the survey form and made an effort to rectify their mistakes before the midterm exam.

4.3c. *Midterm Exam*

The mid term exam was given in the 8th week. Its outcome was used to determine the reliability of Survey II. **115 students** of sections 3, 6, 14 and 19 took the exam. The details of the survey and mid term exam are given in *Appendices 1*(iii)-(iv).

4.4. Survey III & Exit Quiz

4.4a. *Survey III*

By the end of the 13th week, while approaching the end of the course material, the 3rd and final Survey was conducted among the students involved in the experiment directly (i.e, students of Sec. 3, 6, 14 and 19). The objective of the survey was to determine the retention level of students on the course material covered up to the 12th week of the term 042. **100 students** were present during the survey.

4.4b. *Exit Quiz* (*For all the 18 sections which participated in Survey I*)

The students of MATH 102 were given a comprehensive quiz during the 13th week. We shall refer this quiz as to "Exit Quiz". The purpose of this quiz was two-fold. The first objective was to determine the reliability of Survey III which was designed for the students subject to the experiment. The second objective was to determine the difference of understanding among the students of two categories:

(i) Students involved directly in the experiment (**101 students** *sat in the quiz*)

(ii) Students belonging to instructors other than investigators (**172 students** *sat in the quiz*).

In this regard, an analysis of outcomes of both the survey and the exit quiz was carried out. The details of Survey III and the Exit Quiz appear in *Appendices* 1(v)-(vi).

5. Activities during Term 051 (Phase II)

Similar activities as conducted in the term 042 were carried out in Phase II with the same objectives and a similar analysis of surveys and quizzes. Therefore, we shall avoid repetition and briefly provide the description of these activities. It may be noted that students of sections 4, 8 (Dr. M. Bokhari) and sections 12 and 17 (Dr M. Samman) were subject to the experiment.

5.1. Survey I & Related Activities

5.1a. *Survey I* (*Conducted during 1st week of classes*)

Survey I was aimed at determining the awareness of the students on the basic concepts of MATH 001/002 that are frequently used in MATH 101. **559 students** from **15 sections** of MATH 101 participated in the survey. In *Appendix* 2(i), the figure given in a box against each question represents the percentage of students who claimed that they were aware of the concept.

5.1b. *Follow-up Quiz*

The surveyed questions where at least 67% of the students (with a couple of exceptions) claimed their awareness were selected for a Quiz. As before,

- This quiz was given to the students of who participated in Survey I.
- It was conducted during the 1st week of the classes.
- The objective of the quiz was to determine the authenticity of the Survey outcome.

The follow-up Quiz and its outcome may be found in *Appendix* 2(ii).

5.1c. *Preparation of Remedial Material*

The concepts related to Survey I and the Follow-up Quiz where the students' response was poor, were classified as a part of Remedial Material (Same as described in Section 4.1.c). *Appendix* 3(iv) describes the nature of review material.

5.2. Some Teaching Related Activities

5.2a. *Teacher-Student based Learning & Recitation Classes*

For the details, please see Section 4.2.a - 4.2b.

5.2b. *Use of WebCT and Overhead Projector*

Some topics like $(\varepsilon\text{-}\delta)$ definition, differentials, and word-problems based on related rates and maxima/minima consume a lot of lecture time when explained on the chalk-board in calculus classes. In our view, it is wastage of class time to explain complex concepts involving several notations or writing statements of lengthy word-problems. The students made positive comments after receiving explanation of such topics through the overhead projector in classrooms. The students in some cases were asked to look into the displayed material and attempt relevant problems in the class. To provide additional help, the same material was posted on the WebCT.

5.3. Survey II & Midterm Exam

5.3a. *Survey II* (*Conducted during 7th week of classes*)

Like in the case of MATH 102, this survey was also meant only for the sections involved in the experiment directly (See Section 4.3.a for explanation). **93 students** were present at the time of survey. The outcomes of Survey II and Midterm Exams are given in *Appendices 2*(iii)-(iv).

5.3b. *Mid Term Exam*

The mid term exam was given in the 8th week. Its outcome was used to determine the reliability of Survey II. **109 students** appeared in the exam.

5.4. Survey III & Exit Quiz

5.4a. *Survey III* (*Conducted during 13th week of classes*)

As already described in Section 4.4.a, this survey was conducted among the students of the 4 sections with the Project Team. 100 students were present at time the survey. It was designed to determine the retention level of the students on the course material they covered up to the 12th week of classes.

5.4b. *Exit Quiz* (*Conducted during 13th week of classes*)

273 students (101 subject to experiment and 172 from other sections) sat in the Exit Quiz. The details of the Survey III and Exit Quiz are given in *Appendices 2*(v)-(vi).

6. Conclusion I

(*Deficiency & Retention*)

It is important to note that our items selected for the surveys and quizzes (both follow-up and exit) were mostly based on a mono concept. After analyzing the data collected from these surveys, quizzes, midterm exams and some other classroom activities during the course of our experiment, we came across several observations concerning the performance of the students in the ***PCC***. In this regard we shall focus on the following factors:

- Deficient concepts
- Consistent deficiency
- Retention level
- Authenticity of the surveys
- Study habits

6.1. Some definitions/terminologies

In our analysis, we shall define a "*Deficient Concept*" and make a distinction among the levels of deficiency. The concept of "*Consistent Deficiency*" will also be introduced here. In addition, we shall explain what we mean by the "*Retention Level of the Students*" in the context of Calculus I/II.

Definition 1. We shall characterize a concept as a ***Deficient Concept*** if either one of the following criterions holds:

i. the percentage of students that claim to know the concept is less than 75 on Survey I
ii. the percentage of students who attempted a surveyed concept correctly in the follow-up quiz is less than 75

Definition 2. The level of deficiency on a concept is based on its percentage value in Survey I (MATH 101/MATH 102) or in the follow-up quiz I (MATH 101/MATH 102), and is defined as follows:

1. ***High level deficiency*** if the percentage is < 50

2. ***Medium level deficiency*** if the percentage is between 50 and 64
3. ***Low level deficiency*** if the percentage is between 65 and 79.

Definition 3. Any deficient concept which appears more than once in Survey I (MATH 101/MATH 102) or the follow-up quiz I (MATH 101/MATH 102) will be termed as a "***Consistent Deficient Concept***".

Definition 4. The retention level of a course item will be based on its percentage value on "Survey II and Midterm Exam" or "Survey III and Exit Quiz". We shall identify the ***retention level*** as

1. ***High*** if the outcome percentage is > 75
2. ***Medium*** if the outcome percentage is between 65 and 75
3. ***Low*** if outcome percentage is between 50 and 64
4. ***Very Low*** if the outcome is < 50

It may be worthwhile to differentiate between "***Deficiency***" and ***"Retention"*** in our discussion:

- We shall say that a student has a deficiency in a concept while taking a course XXX if the concept is related to one of its prerequisite course(s).
- We shall say that a student has a low level retention in a concept while taking a course XXX if the concept is directly related to the same course.

6.2. Deficient concepts when entering Calculus I class

The material covered by the students in the pre-calculus courses at KFUPM plays an important role in the study of Calculus. Therefore, first we focus on the students' aptitude for these concepts at the time of entering Calculus I course (MATH 101). An analysis based on Definitions 1 and 2 (Sec. 6.1) is summarized in **Table 6.1**.

Table 6.1: **Deficient concepts when entering Calculus I class (Term 051)**

#	Concept	Level of Deficiency		
		Low	Medium	High
1	Simplification involving Laws of Exponents		X *	
2	Simplification of Complex Rational Expressions		X *	
3	Use of Trig. Identities (tan, sec)			X *
4	Use of Trig. Identities (Double Angle)	X *		
5	Use of Trig. Identities (Half Angle)			X *
6	Use of Trig. Identities (csc ←→ sec)		X	
7	Use of Trig. Identities (tan ($a - b$))			X *
8	Use of Rt. Angled Triangle for finding Trig. Relations			X
9	Writing Quadratic Expression in the Square Form		X	
10	Long Division	X		
11	Graph of Absolute Value Function (Shifted $\|x\|$)			X *
12	Graph of Quadratic Function (Shifted x^2)			X *
13	Graphs related to Shift of x^3		X	
14	Graphs related to Shift of $\sqrt{x}$ and ln(x)		X	
15	Graph of e^{-x}		X	
16	Graph of csc x & sec x (Intervals of Increae/Decrease)		X *	
17	Factorization using the formula for $a^3 - b^3$		X *	
18	Determining Coefficients by Expanding $(a - b)^3$		X *	
19	Factorization of Quadratic Like Expressions		X *	
20	Factorization by Pairing		X	
21	Solving Equations which involve $\sqrt{x}$		X	
22	Slope of Line from Two Points on Graph of Function	X *		
23	Distinguishing between Function and Equation			X
24	Domain of Rational Function (Quadratic Denominator)		X	
25	Synthetic Division			X
26	Behavior of Polynomial at ±∞			X
27	Domain and Range of Trig Functions (tan, csc)			X *
28	Domain and Range of $\sqrt{x-1}$		X	
29	Domain and Range of Inverse Trig. Functions	X		
30	Difference between Radian and Degree			X
31	Solution of System of Equations (1 Quad, 1 Linear)		X	
32	Area and Volume of Standard Geometrical Figures			X
33	Distance between Two Points on Graph of Function			X
34	Determining Vertical Asymptotes		X	
35	Determining Horizontal Asymptotes			X
36	Determining Holes in the Graph of given Rational Fun.			X

Legend:

- X represents the deficiency level according to Survey.
- X * represents the deficiency level tested through a follow-up quiz for a concept for which at least 70% of the students' showed their awareness.

6.3. Deficient concepts when entering Calculus II class & Consistent Deficiencies

Here, we consider percentage values on the data based on Survey I and the follow-up Quiz I conducted at the start of MATH 102 classes (Term 042). The items on Survey I and follow-up Quiz I were related to the concepts which the students gained in

- Pre-Calculus courses (MATH 001/002),
- Calculus I (MATH 101),

and are required for the material to be studied in Calculus II.

Table 6.2 identifies the concepts where the students were found deficient. The table describes the level of deficiency and also marks it as **consistent** (see Definition 3) in case it is related to pre-calculus courses.

Table 6.2: Deficient concepts when entering Calculus II class

#	Part I: Concepts Studied in Prep-Year Math Courses	Level of Deficiency			Consistent Deficiency
		Low	Medium	High	
1	Simplification involving Laws of Exponents	X *			Yes
2	Simplification of Complex Rational Expressions	X *			Yes
3	Use of Trig. Identities (tan, sec)			X *	Yes
4	Use of Trig. Identities (Double Angle)		X		Yes
5	Use of Trig. Identities (Half Angle)		X		Yes
6	Use of Trig. Identities (csc ←→ sec)			X	Yes
7	Use of Trig. Identities (tan ($a - b$))	X			Yes
8	Use of Rt. Angled Triangle for finding Trig. Relation			X	Yes
9	Writing Quadratic Expression in the Square Form		X		Yes
10	Long Division		X		Yes
11	Graph of Absolute Value Function (Shifted $\lvert x \rvert$)		X		Yes
12	Graph of Quadratic Function (Shifted x^2)	X			Yes
13	Graphs related to Shift of x^3	X			Yes
14	Graphs related to Shift of $\sqrt{x}$ and ln(x)		X		Yes
15	Graph of e^{-x}	X			Yes
16	Factorization using the formula for $a^3 - b^3$	X			Yes
17	Factorization of Quadratic Like Expressions		X		Yes
18	Factorization by Pairing			X	Yes
19	Solving Equations which involve $\sqrt{x}$		X *		Yes
20	Solution of System of Equations (1 Quad, 1 Linear)	X			Yes
21	Area and Volume of Standard Geometrical Figures		X		Yes
22	Distance between Two Points on Graph of Function			X	Yes

Continuation of Table 6.2 (Deficient Concepts)

#	Part II: Concepts Studied in Calculus I	Level of Deficiency			Consistent Deficiency
		Low	Medium	High	
23	Finding Linear Approximation			X	*Not Applicable Here*
24	Finding Eq. of Tangent Line to Graph of a Function			X	
25	Finding Interval where Function Increases/Decreases			X	
26	Application of $\lim_{x\to\infty}(1+\frac{1}{x})=e$		X		
27	Evaluation of limit, e.g., $\lim_{x\to 0}(\frac{e^x-1}{x})$ (use of L' Hopital rule)		X		
28	Evaluation of limit, e.g., $\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x}-\sqrt{x-1}\right)$			X	
29	Evaluation of limit like $\lim_{x\to\infty}\tan^{-1}x$			X	
30	Derivative of Inverse Trig Functions		X		
31	Derivative of General Logarithm Functions	X			
32	Derivative of General Exponential Functions		X *		
33	Derivative of Trig Functions like csc, sec		X *		

6.4. Retention level

In this section, we shall identify the concepts where the students indicated high, medium or low level of retention (see Definition 4). Our observations are based on the outcomes of Survey II, Midterm Exam, Survey III and the Exit Quiz. It may be noted that

- Survey II was followed by the Midterm Exam in both courses MATH 101 and MATH 102. Unlike the structure of follow-up Quiz which was based on Survey I, the choice of items on the midterm exam was not dependent on Survey II.
- Survey III and Exit Quiz were conducted towards the end of both terms 042 and 051. Some of the items on Survey III were selected for the Exit Quiz to check reliability of the students' response. These items were mostly related to non-deficient concepts (according to Definition 1) where at least 70% of the students under experiment showed their awareness in Survey III.

Each of the **Tables 6.3 and 6.4** is comprised of two parts indicating the students' retention levels "*Around Midterm Exam*" and "*Closer to the Final Exam*". These tables respectively refer to the courses MATH 101 and MATH 102.

Table 6.3 (Related to MATH 101 Students)

Retention Level around Midterm Exam and Before Final Exam

#	Concept	Around Midterm Exam				Closer to Final Exam			
		Very Low	**Low**	**Med**	**High**	**Very Low**	**Low**	**Med**	**High**
1	One-sided limit				X *	X*			
2	Squeezing Theorem		X					X	
3	Limit (*Rationalization*)	X *					X		
4	Limit (*Quadratic Factorization*)				X				X
5	Limit (Application of $\lim_{x\to 0}\frac{\sin x}{x}=1$)				X *		X		
6	Limit of Polynomials at infinity				X *				X
7	Limit by $(\varepsilon-\delta)$ definition				X *	X *			
8	Limit of $\tan^{-1}x$ at infinity	X							
9	Limit of rational func. at infinity				X				
10	Limit of composite functions	X							
11	Concept of continuity at a point				X				
12	Continuity of rational func. at *a*		X						
13	Continuity at end pts of interval			X					
14	Removable discontinuity	X *						X	
15	One-sided continuity							X *	
16	Slope of tangent line to gr of func			X		X *			
17	Eq. of tangent line to Gr of Func.			X *					
18	Instantaneous rate of change				X *		X *		
19	Derivatives by definition $(x, x^2\ldots)$			X *					X
20	Derivatives by defin. $(\sqrt{x}, \sin x...)$	X *							
21	Left & Right derivative at a pt		X						
22	Func. with infinite slope at a pt	X							
23	Higher Derivatives				X				X
24	Product & Quotient rules				X				X
25	Derivatives of Trig Func.				X				
26	Chain rule for derivatives				X *			X *	
27	Related rates				X *			X	
28	Local linear approximation			X *				X	
29	Approx. of numbers using Differentials			X *		X *			
30	Concepts of Relative & % Errors			X					
31	Geometrical Formulas (triangle, circle)				X				
32	Geometrical Formulas (Sphere, Cone, Circular Sector)	X							
33	Simple Graphs (line, $\lvert x + 2\rvert\ldots$)				X				
34	Graphing by Shift of simple Func.		X						
35	Word problems (related rates)			X *			X		

Continuation of Table 6.3 (Retention Level of MATH 101 Students)

#	Concept	Around Midterm Exam				Before the Final Exam			
		Very Low	Low	Med	High	Very Low	Low	Med	High
36	Use of l' Hopital Rule ($\frac{0}{0}, \frac{\infty}{\infty}$ form)	Not Applicable Here				X *			
37	Use of l' Hopital Rule ($0^0, 1^\infty$..)					X			
38	Derivative formulas ($\sec^{-1} x, \log_5 x, 3^x \ldots$)					X *			
39	Implicit Differentiation						X		
40	Finding Critical Numbers								X
41	Intervals where a Function increases or decreases								X *
42	Finding Points of Inflection						X		
43	Intervals where a Function is Concave up/down								X
44	Order of Zeros of a Polynomial					X			
45	Finding Vertical Tangent Lines for a Function						X		
46	Finding Cusps of a Function					X			
47	Appl of 1st & 2nd Derivative Tests							X	
48	Finding Oblique Asymptotes					X *			
49	Finding Curvilinear Asymptotes					X			
50	Difference between Relative & Absolute Max/Min						X		
51	Word Problems (Max/Min)					X			
52	Trig Identities (Simple)							X	
53	Trig Identities (half angle; csc to sec…)					X			
54	Formulas $(a - b)^3$, $a^3 - b^3$							X	

Table 6.4 **(Related to MATH 102 Students)**

Retention Level around Midterm Exam and Before Final Exam

#	Concept	Around Midterm Exam				Before the Final Exam			
		Very Low	Low	Med	High	Very Low	Low	Med	High
1	Use of Trig. Identity (tan←→sec)	X *							X
2	Use of trig. Identity (Double Angle)			X					X
3	Use of Trig. Identity (Half Angle)	X					X *		
4	Use of Trig. Identities (csc ←→ sec)	X				X			
5	Complete square in Quadratic Expression			X					
6	Long Division	X *						X	
7	Graph of Linear function				X				
8	Graph: abs. value func with shifts				X				X
9	Graph: Quad. func. with shifts			X					X
10	Graph: Cubic func. with shifts	X							
11	Graph: $\sqrt{x}$ and ln(x) with shifts		X			X			
12	Graph: 1/x with shifts	X							
13	Graph: Exp. func. with shifts	X				X			
14	Factorize: $a^3 - b^3$ (use of formula)				X				
15	Coefficients in $(a - \mathrm{b})^3$				X				
16	Factorize: Quad. expressions			X					
17	Factorization by pairing	X							
18	Intervals where Function increases or decreases				X	X *			
19	Application of $\lim\limits_{x\to\infty}(1+\frac{1}{x})=e$		X					X	
20	Evaluate limit like $\lim\limits_{x\to 0}(\frac{e^x-1}{x})$ (use of l' Hopital rule)		X						
21	Evaluate: (Rationalization) limit like $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x}-\sqrt{x-1}\right)$	X					X		
22	Evaluation of limit like $\lim\limits_{x\to\infty}\tan^{-1}x$.		X						
23	Finding Limit (Appl. of $\lim\limits_{x\to\infty}x\sin\frac{1}{x}=1$)	X				X			
24	Limit of Sequence with (n!)						X		
25	Derivative: Inverse Trig Func.				X		X		
26	Derivative of general log func.		X			X			
27	Derivative of general exp Func.				X			X	

Continuation of Table 6.4 (Retention Level of MATH 102 Students)

#	Concept	Around Midterm Exam				Before the Final Exam			
		Very Low	Low	Med	High	Very Low	Low	Med	High
28	Deriva. of trig func. (csc, sec)				X				
29	Derivative of hyperbolic func						X		
30	Quotient rule		X						
31	Chain rule for derivatives		X						X
32	Partition of an Interval			X				X	
33	Riemann sum		X						
34	Net signed area			X			X		
35	Local quad/cub. approx of fun		X				X		
36	Monotone sequence	X *							
37	Eventually increasing sequence			X				X	
38	Sequence defined recursively	X				X			
39	Sequence of n^{th} partial sum	X				X			
40	Closed form of n^{th} partial sum	X*				X			
41	Conv./diver. of improper integrals				X		X		
42	Solving definite integrals by geometrical formulas		X					X	
43	Closed form of $\sum_{k=1}^{n} k, \sum_{k=1}^{n} k^2, \sum_{k=1}^{n} k^3, \sum_{k=1}^{n} ar^{k-1}$		X				X		
44	Approx. of definite integral by finite number of rectangles		X				X		
45	Basic anti-derivate formulas		X *				X *		
46	Changing limit of integral with substitution method				X *		X		
47	1^{st} and 2^{nd} part of fundamental theorem of calculus			X					X
48	Mean value theorem for integrals			X					
49	Definition of log function by integrals	X							
50	Solving initial value problems	X					X		
51	Finding formula for n^{th} derivative of functions		X					X	
52	Limit of Seq. by l'Hopital rule		X						
53	Difference between Taylor/Maclaurin polyn.				X *				X
54	Difference between Seq./Series		X						X
55	Sum of telescoping series	X *							
56	General term of a sequence				X *		X *		
57	Integral test			X		X *			
58	Divergence test		X *						X
59	p-series test		X *						X
60	Basic comparison test	X							X

Continuation of Table 6.4 (Retention Level of MATH 102 Students)

#	Concept	Around Midterm Exam				Before the Final Exam			
		Very Low	Low	Med	High	Very Low	Low	Med	High
61	Integrals with combination of Functions $\sin^3 x \cos^2 x$….	Not Applicable Here						X *	
62	Integrals (Algebraic substitutions)					X *			
63	Integrals with combination of Functions $\tan^3 x \sec^2 x$….						X *		
64	Check the series for Abs. Conv/Cond Conv/Div						X		
65	Check: Conv by Ratio Test					X *			
66	Check: Conv by Root Test								X
67	Approx of sum of Alternat. Series					X			
68	Formula: Power Series of $(1 - x)^{-1}$, e^x, $\sin x$						X		
69	Power Series representation of functions using (8)					X			
70	Radius & Interval of Conv. of a Power Series					X			
71	Formula for Vol. of Washer					X *			
72	Formula for Vol. of shell						X *		
73	Formula for Surface Area. of Frustum							X	
74	Vol. of solid of revolution about line // to x-axis							X	
75	Vol. of solid of revolution about line // to y-axis							X	
76	Finding Arc length of a func.							X	
77	Surface Area of solid of revolution						X		
78	Surface Area of solid of revolution (Parametric Form)					X			
79	Integration by parts: $x e^x$, $x\cos x$						X		
20	Integration by parts: $e^x \cos x$						X		
81	Integration by parts: $\sin^{-1} x$					X			
82	Simplify $\sqrt{1+[f'(x)]^2}$ when $f(x)$ is given					X			
83	Solution of System of Equations (1 Quad, 1 Linear)					X *			

7. Conclusion II

(*A Comparative Study of the Performance of Subject & Control*)

As stated above, Survey III and the Exit Quiz were conducted by the end of the terms 042 and 051. Survey III was conducted among the students under experiment. However, students from all those sections which participated in Survey I were invited to write the Exit Quiz. The students were asked to declare their section numbers in order to identify the sections under experiment. Here, our purpose was two-fold:

1. To authenticate the response of the students under experiment (High level response on Survey III versus performance on the Exit Quiz)
2. To compare the performance of the students under experiment with those from the other sections.

7.1. Comparative Study in Phase I (Term 042)

This section deals with the performance of the students of MATH 102 towards the end of the academic term.

7.1. a. *Some explanations*

There are certain points which may be helpful to draw some conclusions out of the comparison provided in Table 7.1:

1. Many students consult a solution manual of the text exercises, for the completion of their homework. Keeping in view this aspect, an exclusive weekly home work was designed for the students under experiment during the Term 042. The problems on the homework, though similar (with some exceptions) to the textbook exercises, were not selected from the textbook. This forced the students to work out the solutions by themselves. This is the first time when the project team noticed a higher achievement of students on Class Quizzes as compared to the quizzes preceded by textbook homework problems. It was also observed that the students frequently visited the instructors' offices to discuss homework problems.

2. The students were involved in group work in the recitation classes. Again, special questions were designed for this activity. These questions were posted on the WebCT at least 4 days prior to the respective recitation class. The students were urged to take a "serious look" at these problems and the concerned material covered in the classroom. 50 minutes allocated to recitation class were distributed over various activities which are stated below:

a.	Group work activity on the selective 3-4 problems:	20-25 minutes
b.	Presentation of solutions, one by each group:	20-25 minutes
c.	Explanation by instructor (if required) or A pop Quiz:	5 minutes

3. The data on the participation of students in Survey I, follow-up Quiz, Survey III and the Exit quiz is as follows:

Activity	**# of Students (Experiment)**	**# of Students (Control)**
Survey I	110	306
Follow-up Quiz	99	230
Survey III	100	None
Exit Quiz	101	172

7.1. b. *Summary of comparative study*

Table 7.1 on the next page provides a summary of comparative performance of the students under experiment (E) versus the "Control"(C) students.

Table 7.1: MATH 102 Students Term 042

Comparative Study of the Performance of Subjects & Control by End of Term

Q. # in Quiz	Item # in Survey	Concept	Survey score (E) (%)	Quiz score (E) (%)	Quiz score (C) (%)
1	7c	Derivative of General Exponential Func :(4^x)	**84**	67	74
2	10	Use of geometric formula, to find the value of $\int_{-2}^{0} \sqrt{4-x^2}\,dx$	**74**	36	41
3	14g	Use of Anti-derivative formulas:: $(v^2+1)^{-1}$	**82**	79	94
4	17a	Solving Integrals by substitution: $\int_0^1 9x^2\sqrt{1-x^3}\,dx$	**85**	**49**	44
5	17b	Integrals of type $\int 4\cos x \sin^3 x dx$	**78**	65	78
6	23	Checking a sequence for Increasing/Decreasing	**70**	**54**	38
7	30a	Use of the Integral Test for testing series	**78**	40	43
8	22	Finding the n^{th} term of the sequence	**83**	**64**	60
9	36b	Volume of Cylindrical Shell (given in the *xy*-plane)	**84**	**54**	12
10	40	Integrals of type $\int_0^{\pi} w \sin w dw$ (Integ. By Parts)	**69**	***60***	***62***
11	4b	Finding the Vertex of parabola ($y = 3 + 2x + x^2$)	**76**	**39**	11
12	20a	n^{th} Derivative of function, e.g., e^{2x}	**64**	70	90
13	8b	Solving System of 2 Eqs (1 Linear, 1 Quadratic)	**84**	**35**	12
14	38a	Volume of Solid of Revolution (Disc Method)	**67**	**58**	43
15	6c	Finding limit of sequence (Rationalization)	**75**	**52**	45
16	1c	Use of half-angle trigonometric identity while solving integrals.	**60**	59	67
17	30e	Use of the Ratio Test for testing a series	**89**	***40***	***42***
18	39i	Finding the Arc Length of a curve	**63**	**73**	54
19	17d	Integrals of type $\int 6\cot^5 x \csc^2 x dx$	**84**	51	76
20	36a	Volume of Washer (given in the *xy*-plane)	**84**	30	42
Average of Survey III and Exit Quiz			**76.65**	**53.75**	**51.4**

Legend:

E: Students under experiment **C:** Students outside experiment

7.1. c. *Some observations*

1. At the end of the term, (E)-Students' perception (Survey) about the course concepts is significantly higher than their actual performance on the Exit Quiz.

2. The students wrote the Exit Quiz out of the entire course material without proper preparation. The low averages (53.75 & 51.4) may be due to this factor (see Table 7.1).
3. (E)-Students on the average basis performed better than (C)-Students. This difference may be due to following factors:
    a. Special homework problems, indeed, helped the students to improve their conceptual understanding (see 7.1.a item 1).
    b. The design of recitation class can be an instrumental factor for slightly better performance of (E)-Students (see 7.1.a item 2).
    c. The number of (C)-Students who participated in the Exit Quiz was 75% of those who participated in the follow-up Quiz given at the start of the term 042. This number in our view is quite reasonable as the students at the end of the term are occupied with several academic activities in different courses (see 7.1.a item 3). As the average performance of an individual indicates, several (C)-Students had a poor score on the Exit Quiz.

    Therefore, we may expect a better performance of (E)-students as compared to (C)-students.

7.2. Comparative Study in Phase II (Term 051)

This section deals with the performance of the students of MATH 101 towards the end of the academic term.

7.2. a. *Some explanations*

Like Section 7.1.a, we present certain points which will enable us to draw some conclusions out of the comparison given in Table 7.2:

1. We had a strong observation from the (E)-Students of MATH 102 about the homework problems. They complained about the excessive amount of time which they had to spend in order to solve exclusively designed homework problems. The project team decided to give the (E)-Students of MATH 101 routine text exercises for homework. The underlying idea for the change was to observe the performance of these students with those of (C)-Students of MATH 101 and also

between the two (E)-Groups (i.e., (E)-students of MATH 101 and MATH 102). As a consequence, we observed that the students scored on the homework almost 90% which was significantly higher than their performance on the class quizzes. As mentioned earlier, several students consult the solution manual for homework problems. This practice results in the students not digesting the course concepts properly. Also, the project team noticed less interaction of the MATH 101 (E)-students with their instructors as compared to what they observed with their (E)-students of MATH 102.

2. The activity of recitation classes was kept the same as before (see 7.1.a item 2). However, the students were given the text problems for this activity.

3. The data on the participation of students in Survey I, follow-up Quiz, Survey III and the Exit quiz is as follows:

Activity	**# of Students (Experiment)**	**# of Students (Control)**
Survey I	105	454
Follow-up Quiz	101	282
Survey III	100	None
Exit Quiz	87	141

7.2. b. *Summary of comparative study*

Table 7.2 provides a summary of comparative performance of the students under experiment (E) versus the “Control”(C) students.

Table 7.2: MATH 101 Students (Term 051)

Comparative Study of the Performance of Subjects & Control by End of Term

Q. # in Quiz	**Item # in Survey**	**Concept**	**Survey score (E) (%)**	**Quiz score (E) (%)**	**Quiz score (C) (%)**
1	1b	One sided-Limit	**90**	**44**	35
2	1c	Limit by Rationalization	**79**	39	50
3	5a	Left/Right hand Continuity at a Point	**85**	**67**	54
4	7b	L’Hopital Rule (∞/∞ form: $2^x/x^2$)	**69**	40	48
5	10a	Slope of Tangent Line to the Graph of $f(x)$	**95**	***46***	***48***
6	3	Limit by $(\varepsilon - \delta)$ definition	**85**	***24***	***26***
7	4	Differentials: Estimation of Irrational Numbers	**54**	45	52
8	11b	Derivative of $\log_5(x+8)$	**79**	34	46
9	12e	Chain rule in derivatives of Trig Functions	**95**	75	84
10	23	Absolute maxima/minima of function on $[a,b]$	54	**57**	52
11	21	Horizontal & Vertical Asymptotes	**28**	**14**	10
12	14	Higher Derivatives with Quotient Rule	**79**	***75***	***77***
13	16b	1st Derivative Test to determine intervals where Func is Increasing/Decreasing Property	**90**	**83**	80
14	11d	Chain rule in derivatives of Rational Func	**87**	*67*	*70*
15	16c	Points of Inflection	**85**	***51***	***52***
16	10c	Instantaneous Rate of Change in Functions	**67**	**56**	52
Average of Survey III and Exit Quiz			**76.31**	**51**	**52.25**

Legend:

E: Students under experiment **C:** Students outside experiment

7.2. c. *Some observations*

Besides some similarities, there are some significant differences in the observations made in 7.1. c and those given below:

1. Like the outcome of Survey III for MATH 102 (E)-Students at the end of the term, (E)-Students of MATH 101 showed a similar understanding of the concepts at the time of their third Survey (From Tables 7.1 and 7.2, note the difference between averages). This point clearly highlights that the students do have awareness about a concept but are unable to utilize it in an exam or test (may be due to lack of sufficient practice).

2. The students wrote the Exit Quiz out of the entire course material without proper preparation. The low averages (51.0 & 52.25 as given in Table 7.2) are similar to those observed in Table 7.1.
3. (C)-Students on the average basis slightly performed better than (E)-Students. Also, the performance of the current (E)-Students was not as good as that of (E)-Students of the preceding term 042. Although these students belong to different courses, the level of Exit Quiz was similar. This difference in performance may be due to the following factors:
    a. The (E)-students were not given special homework problems as it was done for (E)-Students of MATH 102.
    b. The (E)-students had a poor performance on Classroom as well as Recitation Class quizzes.
    c. The number of (C)-Students who participated in the Exit Quiz was only 50% of those who participated in the follow-up Quiz (See 7.2 a. item 3). Here, we have a feeling (though not authenticated) that only the good and motivated students of MATH 101 from other sections wrote the Exit Quiz.

8. Statistical Analysis of Data

(*Surveys, Follow-up Quizzes & Midterm Exams*)

Here, we present a statistical analysis of the data based on the Surveys, Follow-up/Exit Quizzes and the Midterm Exams conducted during the period of experiment (Terms 042 & 051).

8.1. Data Analysis related to Term 042. (MATH 102)

This section deals with the analysis related to MATH 102 students. In this regard we shall tabulate the outcome of Quizzes/Midterm Exam items vs. corresponding survey items. Statistical analysis will be provided for each table followed by comments.

8.1. a. (*E*) & (*C*)-*Students at the time of entering MATH 102 Classes*

Table 8.1

Result of Survey I & Follow-up Quiz I				
# in Quiz	# in Survey	Quiz score (%)	Survey score (%)	Difference
1	2	73	86	13
2	16a	90	88	-2
3	13e	94	85	-9
4	4a	80	78	-2
5	13g	80	81	1
6	14c	62	84	22
7	14d	59	75	16
8	14e	91	92	1
9	14f	92	88	-4
10	14g	87	90	3
11	15a	81	87	6
12	8a	87	84	-3
13	9a	71	95	24
14	9b	76	82	6
15	8b	71	92	21
16	10	71	79	8
17	1	72	87	15
18	3a	33	87	54
19	13d	76	89	13
20	8c	75	94	19

Analysis based on Table 8.1	Quiz score (%)	Survey score (%)
Mean	76.0500	86.1500
Variance	196.5763	28.2395
Number Of Observations	20	20
Hypothesized Mean Difference	0	
df	24	
t Stat	-3.01247	
P-value(One-Tail)	0.004592	
P-value(Two-Tail)	0.005965	

Comments: There is a significant statistical difference between the two means. We can therefore conclude that the students claim to know more than what they can actually do.

Note: The test is based on that the two population variances are not equal, because there is a significant difference between them.

8.1. b. (*E*)-*Students of MATH 102 around the Midterm Exam*

Table 8.2

Result of Midterm Exam vs Survey Score

# in Quiz	# in Survey	Quiz score (%)	Survey score (%)	Difference
1	9a	70.4	53	-17.4
2	9d	65	54	-11
3	14	96.2	63	-33.2
4	3	35.7	76	40.3
5	15a	48.4	51	2.6
6a	2a	27.7	65	37.3
6b	14	75	63	-12
6c	14	54.3	63	8.7
6d	3+13	39	74.5	35.5
8	15	84.8	51	-33.8
9	16	85	57	-28
10i	15f	59.2	63	3.8
10ii	15f	62.9	63	0.1
10iii	15f	59	63	4
11	9k	44.9	44	-0.9
12	15i	39.7	41	1.3
13a	17b	57.7	56	-1.7
13b	17c	63.2	70	6.8
13c	17d	32.4	54	21.6
14	15g	79	62	-17
15	15e	59.6	48	-11.6
16a	13	52.4	73	20.6
16b	15	83	51	-32
16c	9f	31.5	65	33.5

Analysis based on Table 8.2	Midterm Exam score (%)	Survey score (%)
Mean	58.58333	59.31250
Variance	360.43101	86.16984
Number Of Observations	24	24
Hypothesized Mean Difference	0	
df	33	
t Stat	-0.169033	
P-value(One-Tail)	0.866511	
P-value(Two-Tail)	0.866790	

Comments: There is NO significant statistical difference between the two means. We can therefore conclude that the students actually know what they claim. It may be noted in this particular case that both Survey II and the Midterm exam were designed for the (E)-students only. Based on the conclusion from the Analysis, it can be stated that the students filled in the Survey II items more seriously as compared to those who took Survey I.

Note: The test is based on that the two population variances are not equal, because there is a significant difference between them.

8.1. c. (*E*)-*Students of MATH 102 towards the end of the Academic Term*

Table 8.3

Exit Quiz vs. Survey III [Only for (E)-Students]

Result of the Exit Quiz vs. Survey III				
# in Quiz	# in Survey	Quiz score (%)	Survey score (%)	Difference
1	6c	67	84	17
2	9	36	74	38
3	13g	79	82	3
4	16a	49	82	33
5	16b	65	86	21
6	22	54	70	16
7	29a	40	78	38
8	21	64	83	19
9	35b	54	84	30
10	39b	60	69	9
11	4b	39	76	37
12	19a	70	64	-6
13	7b	35	84	49
14	37a	58	67	9
15	5c	52	75	23
16	1c	59	60	1
17	29e	40	89	49
18	38i	73	63	-10
19	16d	51	70	19
20	35a	30	84	54

Analysis based on Table 8.3	Quiz score (%)	Survey score (%)
Mean	53.75000	76.20000
Variance	190.7236842	75.01052632
Number Of Observations	20	20
Hypothesized Mean Difference	0	
df	31	
t Stat	-6.15896	
P-value(One-Tail)	0.000000	
P-value(Two-Tail)	0.000001	

Comments: There is a significant statistical difference between the two means. We can therefore conclude that the students claimed to know more than what they actually do. This can be attributed to observation 2 made in section 7.1.c.

Note: The test is based on that the two population variances are not equal, because there is a significant difference between them.

8.1. d. (*E*) & (*C*)-*Students of MATH 102 towards the end of the Academic Term*

Table 8.4

(Comparative Study of the Performance of (E) & (C)-Students)

Result of the Exit Quiz				
# in Quiz	# in Survey	Quiz score (%) (E)-Students	Quiz score (%) (C)-Students	Dfference
1	6c	67	74	7
2	9	36	41	5
3	13g	79	94	15
4	16a	49	44	-5
5	16b	65	78	13
6	22	54	38	-16
7	29a	40	43	3
8	21	64	60	-4
9	36b	54	12	-42
10	39b	60	62	2
11	3b	39	11	-28
12	19a	70	90	20
13	7b	35	12	-23
14	37a	58	43	-15
15	5c	52	45	-7
16	1c	59	67	8
17	29e	40	42	2
18	38i	73	54	-19
19	16d	51	76	25
20	35	30	42	12

Analysis based on Table 4	Quiz score (%) for (E)- Students	Survey score (%) for (C)-Students
Mean	53.75000	51.40000
Variance	190.72368	579.094737
Number Of Observations	20	20
Hypothesized Mean Difference	0	
df	30	
t Stat	0.378781	
P-value(One-Tail)	0.706958	
P-value(Two-Tail)	0.707492	

Comments: There is NO significant statistical difference between the two means. We can therefore conclude from the Analysis that our sample is cross-representative of the student population taking MATH 102.

Note: The test is based on that the two population variances are not equal, because there is a significant difference between them.

8.2. Data Analysis related to Term 051 (MATH 101)

This section deals with the analysis related to MATH 101 students. Like the case of MATH 102, here we shall also tabulate the outcome of Quizzes/Midterm Exam items vs. corresponding survey items. Statistical analysis will be provided for each table that will be followed by the comments.

8.2. a. (*E*) & (*C*)-*Students at the time of entering MATH 101 Classes*

Table 8.5

Result of the Follow-up Quiz-101				
# in Quiz	# in Survey	Quiz score (%)	Survey score (%)	Difference
1	1	61	86	25
2	2	64	86	22
3	3a	38	92	54
4	3b	69	89	20
5	3c	44	86	42
6	3e	49	87	38
7	4a	71	66	-5
8	8a	78	90	12
9	8b	44	92	48
10	8d	47	73	26
11	8j	51	70	19
12	9a	55	89	34
13	9b	56	80	24
14	9c	56	75	19
15	11	70	77	7
16	16a	29	81	52
17	16b	44	80	36
18	17	69	74	5
19	22a	69	89	20

Analysis based on Table 5	Quiz - I	Survey-I
Mean	56.0000	82.2105
Variance	174.5556	61.7310
Number Of Observations	19	19
Hypothesized Mean Difference	0	
df	29	
t Stat	-7.4300	
P-value(One-Tail)	0.0000	
P-value(Two-Tail)	0.0000	

Comments: There is a significant statistical difference between the two means. We can therefore conclude that the students claim to know more than what they can actually do. The same observation was made for Survey I and the follow-up Quiz in the case of MATH 102.

Note: The test is based on that the two population variances are not equal, because there is a significant difference between them.

8.1. b. (E)-Students of MATH 101 around the Midterm Exam

Table 8.6

Result of Midterm Exam vs. Survey Score

# in Midterm	# in Survey	Midterm score %	Survey score (%)	Difference
1	1iv	91	96	5
3	2e	84	83.1	-0.9
2	2g	83	94.1	11.1
5	3	62	81.5	19.5
6	4v	47	44.4	-2.6
8	4vii	61	71.7	10.7
10	4ix	77	75.8	-1.2
7	4x	78	70	-8
9	4xviii	81	87.2	6.2
11	4xix	44	75.1	31.1
4	5ii	84	45.3	-38.7
12	6i	65	64.6	-0.4
13	6v	47	64.2	17.2
61	9a	55	61.1	6.1

Analysis based on Table 8.6	Midterm score %	Survey score (%)
Mean	68.50000	72.43571
Variance	256.42308	249.49632
Number Of Observations	14	14
Hypothesized Mean Difference	0	
df	26	
t Stat	-0.654707	
P-value(One-Tail)	0.518407	
P-value(Two-Tail)	0.5184085	

Comments: There is no significant statistical difference between the two means. We can therefore conclude that the students actually know what they claim. It may be noted in this particular case that both Survey II and the Midterm exam were designed for the (E)-students only. Based on the conclusion from the Analysis, it can be stated that the students filled in the Survey II items more seriously as compared to those who took Survey I.

Note: The test for the equality of the two population variances shows that there is no significant difference.

8.2. c. (E)-Students of MATH 101 towards the end of the Academic Term

Table 8.7

Exit Quiz vs. Survey III [Only for (E)-Students]

Result of the Exit Quiz vs. Survey III				
# in Quiz	# in Survey	Quiz score (%)	Survey score (%)	Difference
1	1b	44	90	46
2	1c	39	79	40
3	5a	67	85	18
4	7b	40	69	29
5	10a	46	95	49
6	3	24	85	61
7	4	45	54	9
8	11b	34	79	45
9	12e	75	95	20
10	23	57	54	-3
11	21	14	28	14
12	14	75	79	4
13	16b	83	90	7
14	11d	67	87	20
15	16c	51	85	34
16	10c	56	67	11

Analysis based on Table 8.7	Quiz score (%)	Survey score (%)
Mean	51.06250	76.31250
Variance	367.39583	327.02917
Number Of Observations	16	16
Hypothesized Mean Difference	0	
df	30	
t Stat	-3.83273	
P-value(One-Tail)	0.000603	
P-value(Two-Tail)	0.000606	

Comments: There is a significant statistical difference between the two means. We can therefore conclude that the students claimed to know more than what they actually do. This can be attributed to observation 2 made in section 7.2.c.

Note: The test for the equality of the two population variances shows that there is no significant difference.

8.2. d. (E) & (C)-Students of MATH 101 towards the end of the Academic Term

Table 8.8

(Comparative Study of the Performance of (E) & (C)-Students)

Result of the Exit Quiz				
# in Quiz	# in Survey	Quiz score (%) (E)-Students	Quiz score (%) (C)-Students	Difference
1	1b	44	35	8
2	1c	39	50	-11
3	5a	67	54	13
4	7b	40	48	-7
5	10a	46	48	-2
6	3	24	26	-2
7	4	45	52	-7
8	11b	34	46	-12
9	12e	75	84	-9
10	23	57	52	6
11	21	14	10	4
12	14	75	77	-3
13	16b	83	80	3
14	11d	67	70	-3
15	16c	51	52	-1
16	10c	56	52	4

Analysis based on Table 8.8	Quiz score (%)	Survey score (%)
Mean	51.00575	52.26064
Variance	363.21399	368.30600
Number Of Observations	16	16
Hypothesized Mean Difference	0	
df	30	
t Stat	-0.185589	
P-value(One-Tail)	0.854016	
P-value(Two-Tail)	0.854016	

Comments: There is no significant statistical difference between the two means. We can therefore conclude from the Analysis that our sample is cross-representative of the student population taking MATH 101.

Note: The test for the equality of the two population variances shows that there is no significant difference.

8.3. Summary of the finding in the statistical analysis

- Students seem to be unaware that their background is weak in some areas of the prerequisite courses. This has been indicated in four of the six surveys and their respective follow-up and Exit Quizzes (Survey I and its Follow-up Quiz, Survey III and the Exit Quiz for MATH 101 and MATH 102).
- Similarly, we compared the result of the control group (those who are not in the experiment) and the experimental group in the Exit Quizzes. Here the result has shown that the performance of the Control group and the Experimental group is NOT statistically different. This means that the sample students are a good cross-representative of the students for both MATH 101 and MATH 102. It can also be concluded from the Analysis of the Exit Quizzes that there is no statistically significant difference between retention level of the Experimental group and the Control group.

9. Recommendations

We feel responsible for what we bring in as an outcome of our experiment here. The recommendations we are making in the context of PCC courses may be found unusual. To us, the students entering in the PCC courses are not too bad; however, the current status of the PCC is not up to the level of our satisfaction. In order to clarify the situation, we stress that the students we are letting in KFUPM are not imprudent in general. Their level of understanding in the Calculus Courses is not as bad as it appears on the surveys and quizzes given in this experiment-based project. At the same time, the Department of Mathematical Sciences at KFUPM has got an excellent contingent of faculty that is capable of taking any teaching assignment whether based on conventional or innovative methodology. Here, among the students entering KFUPM, we have to make a distinction between students that are weak in terms of understanding and those in terms of having problems while studying in the new environment. In our opinion, the latter class of students is quite wide and faces problems of a diversified nature which vary from being away from their homes to study related matters that include:

- study habits,
- classroom environment,
- learning process,
- time management,
- process of continuous evaluation.

This is absolutely an exclusive area where the University administration is already taking measures for its improvement. On the other hand, regardless of the nature of the students' deficiencies, if we, the custodians of the academic set-up~~s~~, do not rectify the basic conceptual drawbacks of the students and are unable to give them a lead for good study habits besides other points, the purpose of the entire PCC courses that act like a vertebral column for various disciplines of engineering, computer and physical sciences will be ditched. Below we provide our vision on the subject matter which is based mostly on the current experiment and partly on our past experience.

9.1. Pre-Calculus Courses (Prep-Year Level)

Based on the data collected from Survey I and the Follow-up Quiz (Both in Phase I and Phase II), we note that

1. The students at the time of entering the course Calculus I (MATH 101) carry several deficiencies which are related to Pre-Calculus material, though covered thoroughly at the prep-level at KFUPM. We have further noted during the course of experiment that most of the observed deficiencies remain with the students even up to the time of entering the Calculus II classes (See Sec. 6.3 & Table 6.2 for Consistent Deficiencies). In the presence of such weaknesses, we cannot expect the students to perform well in the Calculus Courses.

   Recommendation I: *The KFUPM Prep-Year Math Program may be suggested to look into the data provided in Table 6.1 and relevant parts of Tables 6.2 and 6.3. In case the concerned body undertakes this exercise, they should also be requested to review the level of items selected for Survey I (MATH 101) and in the relevant Follow-up Quiz and bring these points to the attention of instructors.*

2. In general, several students at the Prep-Year pay ~~a~~ little attention towards an appropriate understanding of basic concepts. This is clearly evident from the low average on the Follow-up Quiz (MATH 101). Keeping in view the level of the Quiz-questions which were mostly based on elementary as well as mono concepts, we expect that the students entering the Calculus I course must attain an average of 75-80% on this type of quizzes. Otherwise, they will face consistent adversity in their calculus classes. It may be noted that a large number of students suffer from the language problem at the time of entering the Prep-Year Math Program although they have taken a similar material on Algebra and Trigonometry though in Arabic during their high school studies. In our opinion, we should not let the students waste what they learnt earlier. Therefore, an instructional strategy should be devised that may help the students use their high school knowledge in the Prep-Year Math courses.

Recommendation II: *There is a need to review the teaching methodologies adopted in the Prep-Year. It is suggested that maintaining English as the language of instruction, Arabic translation of terminologies be introduced during the course of teaching MATH 001. Then the students should be invited to recall the concept on the basis of Arabic terminology. These practices will not only help the students utilize their high school knowledge but also rectify their misconceptions, if any, which they carry on from their high school studies. Secondly, keeping in view the nature of deficiencies of the students, it is not necessary to follow the stream of material provided in the standard texts on Algebra & Trigonometry. In our opinion, the students should be raised from the level they enter the Prep-Year Math Program. It is suggested that the coverage of MATH 001 material should be subdivided into four-five levels that involve frequent visits of basic concepts already introduced in the course. As a specimen, we present a design of course coverage for MATH 001 which may be considered for implementation (with appropriate modification if necessary):*

***Level 1*: (New terminology should be described with its Arabic Equivalence in the class)**

Even & odd integers, Rational numbers, Laws of exponents (with examples involving integer exponents only), Simple linear and quadratic equations, Quadratic formula, Linear inequalities and their graphs, Multiplication and long division of algebraic expressions, Function (Domain, Range), Graphs of $f(x) = x$ and $f(x) = x^2$, $f(x) = \sqrt{x}$ and their behavior in (0,1) and outside (0,1), Graph of $f(x) = |x|$, Geometrical shapes with related terminologies and formulas (Area, perimeter, angles): Circle, different forms of triangle and quadrilateral, regular pentagon, trapezium, Rectangular coordinate system, Equation of line (slope, intercepts), Geometrical meaning of slope when it is between 0 and 1, greater than 1, between –1 and 0, less than –1.

***Level 2*: (New terminology should be described with its Arabic Equivalence in the class)**

Prime and composite numbers, Irrational numbers, Real numbers system, Complex numbers, Sets with examples based on number system, intervals, Quadratic and cubic polynomials, Difference between polynomial expression and equation, Synthetic division and its application in finding second factor when one linear factor is known, Simple factorization by pairing, Algebraic formulas, Rationalization of numbers, Completion of squares and its use in factorization, Vertical & horizontal shifts and reflection of graphs related to $f(x) = x$, $f(x) = x^2$ and $f(x) = | x |$, Relation obtained from the geometrical figures (Word problems based on geometrical figures), Volume and surface area of standard geometrical figures: Cuboids, Cylinders, Spheres, Cones.

Level 3: (New terminology should be described with its Arabic Equivalence in the class)

Laws of exponents (with examples involving rational exponents); Square root and radicands, Operations on sets (Union, intersection, difference), Rationalization of algebraic expressions, Quadratic inequalities and their solutions in set or interval form, polynomial equations and inequalities with absolute value, Slope of line, Parallel and perpendicular lines, Comparative study of the graphs of $f(x) = \sqrt[n]{x}$ and $f(x) = x^n, n = 2, 3, 4.$ Symmetries of function (about y-axis and origin), Domain, range and graphs of $f(x) = \frac{1}{x^n}, n = 1, 2, 3, 4.$ Operations on functions with the help of known examples, Composition of functions. Rational zeros theorem for polynomials, Simple rational functions, Behavior of a polynomial at infinity, Horizontal asymptotes.

Level 4: (New terminology should be described with its Arabic Equivalence in the class)

Word problems based on linear and quadratic equations, Sketching graphs of the form $f(x) = \pm(x+a)^n + b$; $f(x) = \pm\sqrt[n]{x+a} + b$; $f(x) = \pm(x+a)^{-n} + b$, $n = 2, 3, 4, 5$, by using horizontal/vertical shifts and/or reflections. Equation of circle (shift of centre), Symmetries of graph of an equation (about x, y-axes and origin) with simple examples, Algebra of complex numbers, Discriminant of quadratic equation and its properties, Complex roots theorem, Fundamental theorem of Algebra, Sketching graphs of simple rational functions (with linear denominator, Vertical Asymptotes.

Level 5**: (New terminology should be described with its Arabic Equivalence in the class)** Evaluation of difference-quotient for various functions, Variation, Simple mathematical models (constructing functions), Oblique, quadratic and cubic asymptotes, Rational functions and their graphs using all features learnt in the course.

3. The texts on Algebra & Trigonometry accompanied with test banks in which the test items assure the students understanding of course concepts. The level of difficulty is also narrated beside each of the test items. The structure of questions given in the test bank usually backs up the Calculus material and is not unnecessarily twisted in nature. The students doing well on the tests based on such items or alike do not possess any deficiency as we have observed in the outcomes of follow-up quiz given to MATH 101 or MATH 102 students in our experiment.

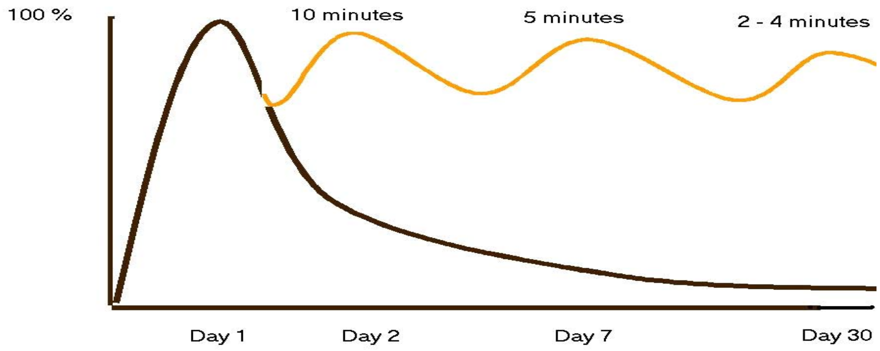
   Recommendation III: *It may be appropriate to devise an evaluation policy that may ensure students' comprehension level of basic concepts. We suggest that the style of the exam-questions provided in the Test Bank accompanying the textbook should be followed, and to keep the standard high, the passing marks should be fixed at around 75%.*

9.2. Calculus Courses

Based on the data collected from Survey III and the Exit Quiz (in Phases I & II) we note that students' retention level of basic Calculus concepts is not up to the desired level. Although most of the students are found attentive during the Calculus lectures, they keep on forgetting what they learn in the classroom environment. To us, this aspect is mostly related to the students' study habits as well as their deficiency on the Pre-Calculus material which, in some cases, leads to consistent deficiency. The latter part of the problem has already been addressed in the foregoing paragraphs. As far as the first point is concerned, a collaborative effort is needed from both the teacher and the taught. Here, we highlight some points that may enable the students to have a sound grip on the basic calculus concepts. It will be followed by some tips particularly for the new Calculus instructors which may help improving the classroom learning environment.

1. It may be noted that the retention problem is a universal phenomenon and it is widely addressed in the literature as pointed out in Chapter 2. Here, it is also interesting to observe the following figure downloaded from internet resources. It indicates the rate of forgetting a new concept with a possible remedy:

The Curve of Forgetting (*Black Wave*)



Yellow Wave

(*Revision Time & its Frequency to Keep the Material Alive in your Mind*)

This figure clearly stresses on the revisit of new material for the sake of its retention in a systematic way. Thus, a straightforward remedy to this problem is revising a new concept periodically, in particular, by seeking its application to other problems. A natural and reasonable question arising here is that how far a student may go for revision of an old concept in the light of the above figure? Based on our interview with several students, it turned out that most of the students review the material near an exam or sometimes for a quiz. The underlying reason is obvious: either they are deficient in time management or they are overburdened with their academic load. In both cases, they are unable to carry out the review as suggested in the above figure. However, our students appreciated the strategy of mixing the problems related to past and current exercises for the weekly homework.

Recommendation IV: *To cope with this situation, the prep-year material where the students are found deficient should be made available to the students on the WebCT or instructor's homepage. Besides, the instructors may pay additional attention while designing a Calculus course for classroom presentation. In this regard,*

- *Some examples, which involve a couple of concepts covered in the past lectures, should be picked up periodically in the classroom.*
- *1-2 problems based on some of the past concepts should be exclusively designed for a weekly homework* (*see Appendix* 3(ix) *where homework includes some exercises based on past material for the sake of review*).
- *A short question concerning one of the past concepts may be included in a classroom Quiz. However, the students may be asked to review the relevant concepts at hand. The practice will remind the students about the importance of going over the past concepts off and on.*

2. An appropriate design of quizzes and exams forces the students to pay enough attention on the basic concepts.

   Recommendation V: *A part of the quiz and exam should be based on mono concept questions. We have experienced a positive impact on the retention level of students where this strategy is applied.*

3. The students should be indicated clearly what they have to understand and which concept they have to revise to the extent that it should be retained in their mind. Most of the time the students memorize a formula but they are unable to apply it appropriately.

Recommendation VI: *It may be appropriate to post on the WebCT or hand over to the students a list of closely related formulas or specific techniques required in the course related exercises. Each formula should be appended with a couple of exercises* (as an example see *Appendix* 3(v-vi): the list of integration techniques; basic geometrical formulas related to application of integrals.)

4. Reorganization of course topics in contrast to the flow of the text material sometimes helps the students to retain difficult concepts. In particular, if the material having little connection with the preceding course concepts appears at the end of the course, most of the students are unable to digest it. According to our observation, most of the students felt overburdened at the end of the term and therefore, could not pay attention to new concepts at that time. One such example that we encountered in MATH 102 is related to the entire chapter of "Sequences and Series". This portion is usually covered during the last five weeks of the term. Here, the final part which provides the crux of the entire topic "Approximation of Integrals by Power Series Representation" is covered around the second last lecture of the course. Therefore, a vast majority of the students are unable to digest this concept. In our opinion, only a few students retain these concepts after the completion of this course.

Recommendation VII: *To cope with this situation, we strongly suggest reordering some sections or even a chapter of the course according to its nature. This practice may let the students have a thorough review of such topics during the term. Sometimes the reshuffling requires certain uncovered concepts of the course material. However, an instructor can easily find a way to bridge the gap.* (*See reshuffling of contents in the Syllabus of MATH 102 in Appendix* 3*(vii). Here, the entire topic of sequences & series is introduced in the second quarter of the course. This change requires only a couple of concepts like elementary treatment of improper integrals of the first kind and the integration by parts which can be easily introduced to the students at hand. In our view, this change helps the students to grasp this material which they find very much confusing* *when offered at the end of the term. The left over course material after covering this particular chapter is found relatively easier for the students*).

5. Introducing Word Problem in the Calculus courses is another major task for the instructors. Mathematics in general and Calculus in particular turns out not only meaningless but also boring for the engineering, computer and business students if they do not find applications of the subject in their disciplines. However, this activity consumes a lot of class time if the material is not prepared in a "presentable form".

   Recommendation VIII: *The students should be referred to the guidelines given in the text sections for solving the word problems. Alternatively, one-sheet instructions on the subject matter (in simple English) may be posted on the WebCT or the instructor's homepage. Writing the statement of word problems on the board should be avoided. Instead, overhead projectors or power point presentation should be used for this purpose. Use of tables to complete the required steps is found useful in this context.* (See *Appendix* 3(viii) for introducing the "Applications of Maxima/Minima" in MATH 101)

6. The weekly recitation classes meant for problem-solving in Calculus I & II (MATH 101 and MATH 102) do help the students in increasing their understanding of the subject and retention level if organized properly. The students may benefit a lot from these classes when given an opportunity to solve problems in a supervised environment.

   Recommendation IX: *The problem solving classes (Weekly Recitation Class) should be designed in such a way that reflects maximum amount of students' participation. In these classes we may:*

   - *generate a group activity which is based on solving pre-assigned problems,*
   - *ask students to explain solutions on the board,*

- *introduce challenging problems,*
- *discuss the common mistakes/problems faced by the students,*
- *introduce and maintain use of technology, e.g., software like Mathematica,*
- *introduce any other activity which may help the students in increasing their understanding as well as retention level.*

If these classes are being conducted by a faculty/graduate assistant other than the course instructor, a weekly interaction between the two is highly recommended. (See *Appendix* 3(iii) regarding the design of Problem Session Classes carried out during the period of experiment. Students found the designed activity helpful for their understanding).

7. Evaluation & Exam Analysis are significant tools to assess the retention level of students. However, designing an exam with this objective requires a lot of effort on the part of a course instructor. In particular, a careful selection of test items is required in order to measure the students' retention level on the essential course concepts. The current research on appropriately designing a Calculus exam suggests that a student regularly attending his classes should be able to solve at least 50% of the exam items correctly. A part of such exam items are in fact referred to as mono concepts related to the course. An itemized analysis of such exams provides an insight to an instructor regarding his students' understanding of course concepts.

Recommendation X: *Here we recommend that an instructor should be clear about what he intends to measure from an exam item in the PCC courses. The exam items should be spread uniformly over the course material. Moreover, the exams should be designed to check what the students know about the course and not what they don't. As indicated above, after grading an exam, its item analysis should be given due attention as it provides a way to judge the students' understanding of specific concepts and paves a way for the instructor to improve his teaching abilities.*

8. With the passage of time, technology (computers, overhead projectors, WebCT, internet resources etc.) is becoming an integral part of teaching. In our opinion, an advantage of this facility should be taken as far as its utilization improves the students' comprehension of course material. However, it is important to note that no component of technology can substitute the role of an instructor, at least in the Calculus courses.

   Recommendation XI: *There are various ways to make positive use of technology in and outside the classroom. However, we suggest that the off and on use of an overhead projector and power point should be considered by the instructors in PCC courses. Some of the advantages in this regard are already mentioned above. Also, the use of WebCT or other equivalent tools should be made compulsory in order to interact with the students outside the classrooms. The software like "Mathematica or MATLAB" is a good tool to introduce certain tedious concepts related to the course material and its use motivates the students for further exploration of related ideas outside the text. However, it is should not be done at the expense of regular class timings. An instructor may arrange a part of recitation class or a help session for this purpose. We recommend that such activity should be left at the discretion of the course instructor unless the department formulates a unified policy in this regard.*

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  b. A. J. Salem, “A mathematics technology classroom: Evolution of a calculus reform implementation”.

  c. V. Kilibarda, “Students’retention in the light of calculus reform”.

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11. Appendices

Appendix - I

Analysis of Survey I & Follow-up Quiz I (MATH 102-042)

Appendix 1(i)

الأسئلة الآتية متعلقة بمفاهيم سابقة تحتاجها في دراسة مادة الرياضيات Math 102. الرجاء قراءة كل سؤال بعناية ثم ضع علامة √ داخل المربع المقابل له إذا كنت تعتقد أن بإمكانك حله (لا تحاول حل السؤال)، و إذا كنت تعتقد أنه لا يمكنك حل السؤال فاترك المربع فارغاً هكذا ☐

Participated in: **Survey** **416 Students** **Quiz** **329 Students**

---

1. Do you know the **Laws of Exponents,** e.g.

$$\frac{\left(2^3.2^{-2}\right)^4}{(3.2)^{-3}}$$

87 %   72 %

2. Do you know how to **simplify** complex expressions, e.g.,

$$\frac{2-\frac{3}{x-1}}{x+\frac{2x}{3-x}}$$

86%   73%

3. Do you know the following **Trigonometric Identities**, e.g.

a. $1+\tan^2\theta=\sec^2\theta$ 87%   33%

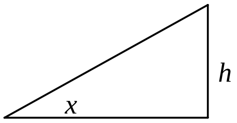
b. $\cos 2\theta=\cos^2\theta-\sin^2\theta$ 59%

c. $\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}$ 51%

d. $\csc\left(x-\frac{\pi}{2}\right)=-\sec x$ 38%

e. $\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}$ 74%

4. Do you know how to use **Right Angled Triangle** to find **Trigonometric Ratios**, e.g. If $\sin x=h$ in the diagram, then



a. $\sec x={}^{1}\!\big/_{\sqrt{1-h^2}}$ 78%   80%

b. $\sin(2x)=2h\sqrt{1-h^2}$ 33%

c. $\cos\frac{x}{2}={}^{\sqrt{1+\sqrt{1-h^2}}}\!\big/_{2}$ 18%

5. Do you know if $x=3\sec\theta$, then

$\sqrt{x^2-9}=3|\tan x|$ 22%

6. Can you find *a, b* and *c* when $5-4x-2x^2$ is written in the **form** $a+b(x+c)^2$?

59%

7. Can you do the **long division**, e.g., divide $5x^4+3x^2-4x-5$ by x^2-2x+4?

65%

8. Can you sketch the **graphs** of following functions?

a. $y=2x-3$ 84%   87%

b. $y=|x+2|$ 92%   71%

c. $y=x^2$ 94%   75%

d. $x=y^2-5y+6$ 76%

e. $y=x^3-6$ 74%

f. $y=\sqrt{x-1}$ 67%

g. $y=\ln(x+1)$ 59%

h. $y=e^{-x}$ 60%

9. Can you **factorize**

a. x^3-64 95%   71%

b. $(x-2)^3$ 82%   76%

c. x^6-5x^3+6 61%

d. $3x^3-x^2+3x-1$?   45%

10. Can you find the **solution** of $\sqrt{9x}=x+2$?

79%   71%

11. Linear approximation of $f(x)=\sin x$ about $x=\frac{\pi}{6}$ is given by $f(x)\approx\frac{1}{2}+\frac{\sqrt{3}}{2}\left(x-\frac{\pi}{6}\right)$. Can you find the equation of the **tangent line** to the graph of $y=\sin x$ at $\frac{\pi}{6}$?

48%

12. Can you find the interval(s) in which the function $f(x)=\frac{-x}{1+x}$ is **decreasing**?

64%

13. Can you find the following **limits**:

a. $\lim_{x\to\infty}\left(1-\frac{2}{3x}\right)^{4x}$ 55%

b. $\lim_{x\to 0}\frac{e^x-1}{x}$. 63%

c. $\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x}-\sqrt{x-1}\right)$ 46%

d. $\lim_{x\to 3}\frac{x-3}{x^2-5x+6}$. 89% 76%

e. $\lim_{x\to\infty}\frac{8x^2-3x-11}{5-3x+7x^2}$ 85% 94%

f. $\lim_{x\to-\infty}\tan^{-1}x$ 41%

g. $\lim_{x\to 0}\frac{\sin 3x}{x}=3$? 81% 80%

14. Can you find the following **derivatives**:

a. $\frac{d}{dx}\sec^{-1}(x+1)$ 63%

b. $\frac{d}{dx}\log_5(x+8)$ 70%

c. $\frac{d}{dx}3^x$ 84% 62%

d. $\frac{d}{dx}\csc(x-5)$ 75% 59%

e. $\frac{d}{dx}\sqrt{x-1}$ 92% 91%

f. $\frac{d}{dx}\left(\frac{x}{x+1}\right)$ 88% 92%

g. $\frac{d}{dx}(x^2-3)^{-8}$ 90% 87%

15. Can you solve the **system of equations**?

a. $5x+3y=0$, $2x-4y=1$ 87% 81%

b. $x^2+y^2=7$, $2x+y=-3$ 72%

16. Do you know the **formulae** related to following **Geometrical figures**:

a. Area of **Triangle** 88% 90%

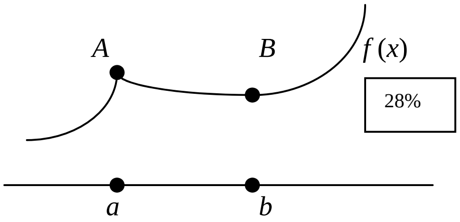
b. Area of **Trapezoid** of height *h*

31%

c. Volume of **Cylinder** 68%

d. Surface area of a **Cone?** 60%

17. Can you find the **distance between points** A and B given in the diagram:



28%

18. Are you aware of the **formulae**:

a. $\sum_{k=1}^{n}k=\frac{n(n-1)}{2}$. 18%

b. $a+ar+ar^2+\cdots+ar^{n-1}=\frac{a(1-r^n)}{1-r}$? 14%

Note: The figure in the white box represents the percentage of students who showed their awareness of the material included in that item. The figure in the **red box** against a **survey item** represents the percentage of students who gave correct answer for the item when included in the **Quiz.**

Name:__________ Quiz (MATH 102) [C1] ID#__________ *Appendix*1(ii)

(Number of Students who wrote the Quiz: 329)

1. $\dfrac{1-\dfrac{3}{x-1}}{4+\dfrac{2x}{2-x}}=$

73%

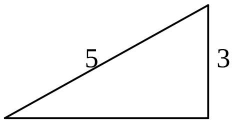
a. $\dfrac{2-x}{x-1}$

b. $\dfrac{x-2}{2(x-1)}$

c. $\dfrac{x-2}{x-1}$

d. $\dfrac{2-x}{3(x-1)}$

2. Area of the given **Triangle** is



90%

a. 15
b. 6
c. 8
d. $\dfrac{15}{2}$

3. $\lim\limits_{x\to\infty}\dfrac{5x-2x^3+11}{7x^3+5x^2-2}=$

94%

a. ∞

b. $\dfrac{5}{7}$

c. $\dfrac{-2}{7}$.

d. $-\dfrac{11}{2}$

4. If $\tan x=\dfrac{1}{h}$, then $\cos x=$

80%

a. $\dfrac{h}{\sqrt{h^2+1}}$

b. $\dfrac{\sqrt{1+h^2}}{\sqrt{1-h^2}}$

c. $\dfrac{1}{\sqrt{1+h^2}}$

d. $\sqrt{1+h^2}$

5. $\lim_{x\to 0}\frac{\sin 3x}{5x} =$

a. $\frac{5}{3}$

80%

b. $\frac{3}{5}$

c. 1

d. does not exist.

6. The value of $\frac{d}{dx}(5^x)$ at $x = 0$ is

a. 5

b. 5ln5

62%

c. ln5

d. 1

7. The value of $\frac{d}{dx}[\csc(x-\frac{\pi}{2})]$ at $x = \pi$ is

a. 0

b. 1

c. -1

59%

d. $\frac{3}{2}$

8. The value of $\frac{d}{dx}(\sqrt[3]{x-1})$ at $x = 2$ is

a. $\frac{1}{2\sqrt[3]{2}}$

91%

b. $\frac{1}{3}$

c. $-\frac{1}{3}$

d. $\frac{1}{\sqrt[3]{3}}$

9. $\frac{d}{dx}\left(\frac{x}{x+1}\right) =$

a. $\frac{-1}{(x+1)^2}$

92%

b. $\frac{1}{(x+1)^2}$

c. $\frac{-2}{(x+1)^2}$

d. $\frac{2}{(x+1)^2}$

10. The value of $\frac{d}{dx}(x^2-3)^{-8}$ at $x = 2$ is

87%

- a. 1
- b. 32
- c. – 32
- d. – 16

11. If (x, y) is the solution of the system $\begin{matrix} 5x+3y=1 \\ x+2y=3 \end{matrix}$, then $x + y =$

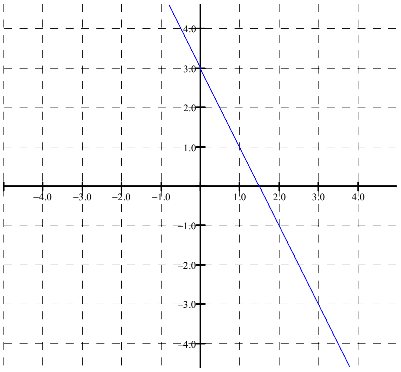
81%

- a. 0
- b. 1
- c. 2
- d. 3

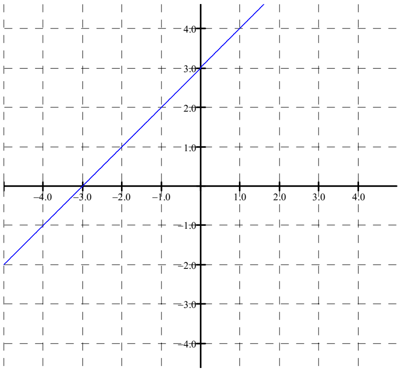
12. The graph of the equation $y=-2x+3$ is:

87%

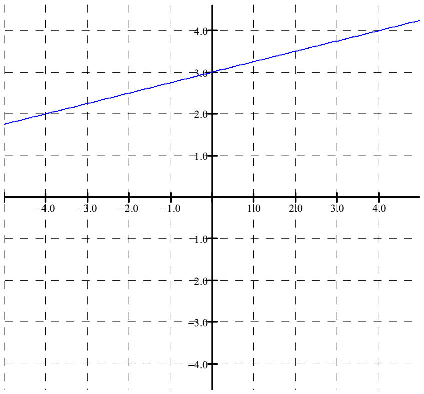
a



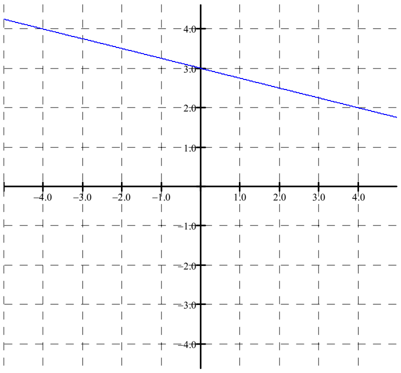
b



c



d



13. One factor of x^3-64 is:

71%

- a. $x^2-8x+16$
- b. $x^2-8x-16$
- c. $x^2+4x+16$
- d. $x^2+4x-16$

14. The coefficient of x^2 in the product $(x-2)^3$ is:

a. -6

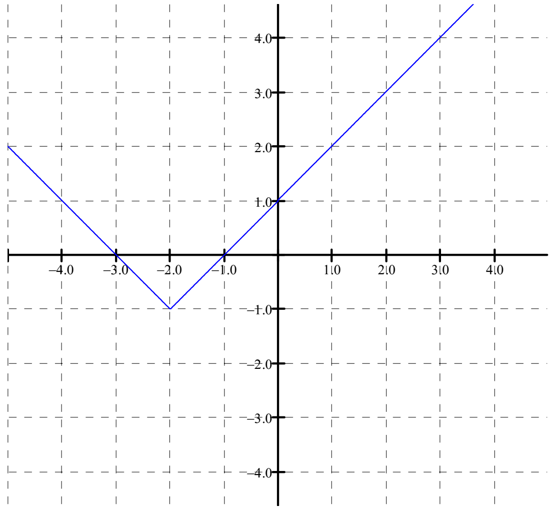
b. 6

c. 12

d. -12

76%

15. The equation that represents the following graph is:



71%

a. $|x+2|+1$

b. $|x-2|+1$

c. $|x-2|-1$

d. $|x+2|-1$

16. The **sum** of the solution set of $\sqrt{9x} = x+2$ is:

71%

$a)\ -5$

$b)\ -3$

$c)\ \ 5$

$d)\ \ 3$

17. $\dfrac{\left(3^5.3^{-2}\right)^{\frac{-1}{2}}}{3^{-1}}$

a. $\sqrt[3]{3}$

b. $3^{\frac{3}{2}}$

c. $\dfrac{1}{\sqrt{3}}$

72%

d. $\dfrac{1}{3}$

18. $1-\sec^2\theta =$

a. $\tan \boldsymbol{\theta}$

b. $\tan^2\boldsymbol{\theta}-1$

c. $\tan^2\boldsymbol{\theta}$

d. $-\dfrac{\sin^2\boldsymbol{\theta}}{\cos^2\boldsymbol{\theta}}$

33%

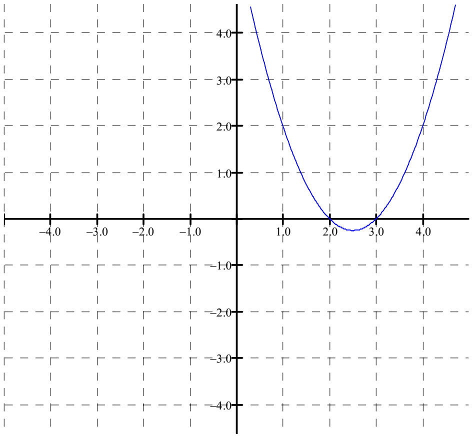
19. $\lim\limits_{x\to 3}\dfrac{x-3}{x^2-7x+12} =$

a. 0
b. 1
c. Does not exist
d. -1

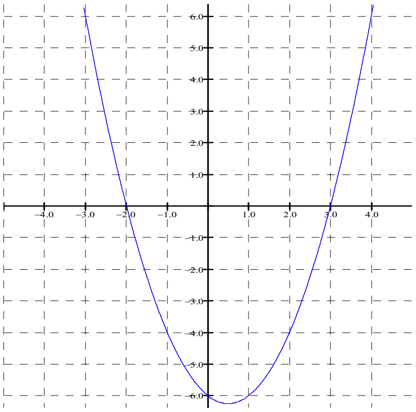
76%

20. The graph of the equation $y = x^2 - 5x + 6$ is:

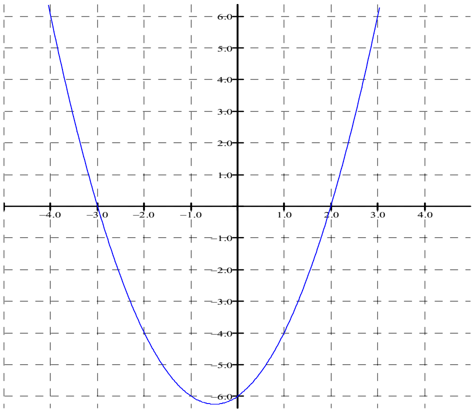
a



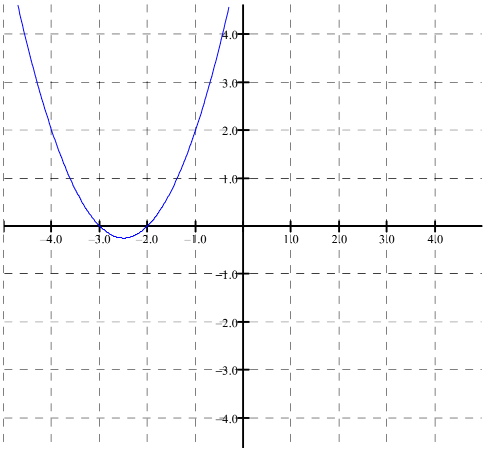
b



c



d



75%

Quiz Overall Average

76%

Analysis of Survey II & Midterm Exam (MATH 102-042)

Appendix 1(iii)

الأسئلة الآتية متعلقة بمفاهيم سابقة تحتاجها في دراسة مادة الرياضيات Math 102. الرجاء قراءة كل سؤال بعناية ثم ضع علامة √ داخل
المربع المقابل له إذا كنت تعتقد أن بإمكانك حله (لا تحاول حل السؤال)، و إذا كنت تعتقد أنه لا يمكنك حل السؤال فاترك المربع فارغاً هكذا ☐

Participated in: **Survey** 100 **Midterm Exam** 115

1.	Do you know the **Laws of Exponents,** e.g. $\frac{\left(2^3.2^{-2}\right)^4}{(3.2)^{-3}}$	90	
2.	Do you know the following **Trigonometric Identities**, e.g.		
a.	$1+\tan^2\theta=\sec^2\theta$	95	27.7
b.	$\cos 2\theta=\cos^2\theta-\sin^2\theta$	65	
c.	$\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}$	47	
d.	$\csc\left(x-\frac{\pi}{2}\right)=-\sec x$	38	
e.	$\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}$	72	
3.	Can you perform the **long division**, e.g., divide $5x^4+3x^2-4x-5$ by x^2-2x+4?	76	37.35
4.	Can you sketch the **graphs** of the following functions?		
a.	$y=2x-3$	96	
b.	$y=\lvert x+2\rvert+4$	86	
c.	$y=(x-2)^2-3$	73	
d.	$y=x^2-5x+6$	63	
e.	$y=(x+1)^3-2$	47	
f.	$y=\sqrt{x-1}+3$	58	
g.	$y=\frac{1}{(x+2)^2}-1$	26	
h.	$y=\ln(x+1)$	55	
i.	$y=e^{-(x+2)+1}$		

5.	Can you **factorize**	20
a.	x^3-64	90
b.	$(x-2)^3$	85
c.	x^6-5x^3+6	66
d.	$3x^3-x^2+3x-1$?	42
6.	Can you find the interval(s) in which the function $f(x)=\frac{-x}{1+x}$ is **decreasing**?	78
7.	Can you find the following **limits**:	
a.	$\lim_{x\to\infty}\left(1+\frac{2}{3x}\right)^{4x}$	50
b.	$\lim_{x\to0}\frac{e^x-1}{x}$.	63
c.	$\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x}-\sqrt{x-1}\right)$	41
d.	$\lim_{x\to\infty}\frac{8x^2-3x-11}{5-3x+7x^2}$	92
e.	$\lim_{x\to-\infty}\tan^{-1}x$	52
f.	$\lim_{x\to\infty}x\sin\frac{2}{5x}$?	45
8.	Can you find the following **derivatives**:	
a.	$\frac{d}{dx}\sec^{-1}(x+1)$	77
b.	$\frac{d}{dx}\log_5(x+8)$	60
c.	$\frac{d}{dx}3^{2(x-1)}$	79

d. $\frac{d}{dx}\csc(x-5)$ 76

e. $\frac{d}{dx}\sqrt{x-1}$ 94

f. $\frac{d}{dx}\left(\frac{x}{x+1}\right)$ 53

g. $\frac{d}{dx}(x^2-3)^{-8}$ 56

9. Are you familiar with the following terms:

a. **Partition** of an interval 53 70.4

b. Integrand 56

c. Riemann Sum 52

d. net signed area 54 65

e. **Local Quadratic or Cubic Approximation** of a function about a point 62

f. Monotone Sequence 65 31

g. **upper bound** of a sequence 72

h. **eventually increasing** sequence 73

i. Sequence defined **recursively** 19

j. Sequence of **nth partial Sum** of an Infinite Series 51 44.9

k. **Closed Form** of nth partial Sum 44

l. **Convergence or Divergence** of improper integrals 77

10. Can you evaluate a definite integral in which the integrand involves basic geometric figures like **part of a Circle**, **Trapezoid**, **rectangle** and **triangle**? 58

11. Can you write the following finite sums in **closed form**: 56

$\sum_{k=1}^{n} k; \sum_{k=1}^{n} k^2; \sum_{k=1}^{n} k^3; \sum_{k=1}^{n} ar^{k-1};$.

12. Can you **approximate an integral** by finite number of rectangles? 57

13. Are you aware of **Basic anti-derivative formulas** 73 52.4

14. Do you know **how to change the limits of integration** when you solve an integral by substitution method? 63 75.33

15. Do you know

a. 1st & 2nd part of the **Fundamental theorem of Calculus**? 72.07

b. Mean Value theorem for Integrals 70

c. definition of **logarithmic function** with the help of integral 38

d. How to solve **initial value problem** 29

e. How to find a **formula for nth Derivative of a function**. 48 59.6

f. How to find **limit of a sequence** using I'Hopital Rule 63 60.37

g. The difference between **Taylor and Maclaurin Polynomials** 62 79

h. Difference between a **sequence and a series** 51

i. How to find the **sum of the series** like $\sum_{n=2}^{\infty}\frac{1}{n^2+n-2}$ 41 39.7

16. Can you write **general term of a sequence** when first 3 or 4 terms are known? 57 85

17. Do you know the following tests related to Infinite series?

a. **Integral** Test 65

b. **Divergence** Test 56 57.7

c. **P-series** test 70 63.2

d. **Basic Comparison** Test 54 32.4

****

Note: The figure in the white box represents the percentage of students who showed their awareness of the material included in that item. The figure in the **red box** against a **survey item** represents the average percentage score obtained by the students while giving the answer related to the item given in the Midterm Exam.

The choice of exam items is not dependant on the survey outcome unlike Survey I

Appendix 1(iv)

Midterm Exam

MATH 102

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

(Mid-Term Exam 042)

Time 120 min

MATH 102

Name:____

ID#____

Serial#:____

Sec.#:____

No calculator is allowed in the exam.

Show all necessary work

1. Partition the interval $[0, 8]$ into 4 equal subintervals and then approximate the integral $\int_0^8 (x + 2)dx$ by using 4 subintervals and mid point rule. (5 points)

2. Find the net signed area between the curve $y = x^2 - 5x + 6$ and the interval $[-3, 3]$. (5 points)

3. If $\displaystyle\int_0^5 f(x)dx = 7$ and $\displaystyle\int_2^5 f(x)dx = -1$ then find $\displaystyle\int_0^2 f(x)dx.$ (3 points)

4. Evaluate $\displaystyle\int_3^5 \frac{x^2+6x-5}{x+5}\,dx.$ (5 points)

5. If $F(x) = \int_1^{x^2} \left(t^2 - 2\sqrt{t}\right) dt$ find $F'(1)$. (3 points)

6. Evaluate each of the following integrals

   (a) $\displaystyle\int \sin^2 x\, dx.$ (3 points)

   (b) $\displaystyle\int x\sqrt{x+1}\, dx.$ (5 points)

   (c) $\displaystyle\int \frac{\ln\sqrt{x}}{x}\, dx.$ (5 points)

   (d) $\displaystyle\int \frac{x^2+2}{x^2+1}\, dx.$ (5 points)

7. Using the substitution $u = 4x^2 - 3$, if we write $\int_{\sqrt{2}}^{5} x(4x^2-3)^{19}dx = \int_{a}^{b} f(u)du$ then find a, b and $f(u)$. (4 points)

8. If $\int f(x)dx = \frac{x}{x+1} + c$ then find $f(x)$. (4 points)

9. Find the n^{th} term of the sequence $-2, 4, -6, 8, -10, ...$ (2 points)

10. Find the limit of the sequence when it exists

(i) $\left\{\left(1+\dfrac{5}{3n}\right)^{-2n}\right\}_{n=1}^{\infty}$ (5 points)

(ii) $\left\{\sqrt{n^4-2n^2+8}-n^2\right\}_{n=1}^{\infty}$ (5 points)

(iii) $\left\{(-1)^n\dfrac{n^2+1}{n^2}\right\}_{n=1}^{\infty}$ (5 points)

11. Find the closed form of the sum

$$\sum_{k=1}^{n} \frac{6^{k-2}}{8^{k+4}}$$

(5 points)

12. Find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{k+2} - \frac{1}{k+5} \right)$. (5 points)

13. Which of the following series converge or diverge. Give reason:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{5n}{2n-6}$ (3 points)

(b) $\displaystyle\sum \frac{1}{\sqrt[3]{n^2}}$ (3 points)

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(2n+1)}{2n+1}$ (5 points)

14. Find the 4th Taylor Polynomial of $\cos x$ about $x = \pi$. (5 points)

15. Find the n^{th} derivative of $f(x) = e^{2(x+5)}$. (4 points)

16. Answer TRUE or FALSE with justification. (2 points each)

(a) For any interval I in $\mathcal{R}$, the function $f(x) = \dfrac{1}{1-x}$ has an antiderivative.

☐

(b) The fundamental theorem of calculus is used in evaluating definite integrals.

☐

(c) If a sequence $\{a_n\}_{n=1}^{\infty}$ is monotone and all its terms lie between the numbers 25 and 1000, then this sequence must diverge.

☐

Analysis of Survey III & Exit Quiz (MATH 102-042)

Appendix 1(v)

الأسئلة الآتية متعلقة بمفاهيم سابقة تحتاجها في دراسة مادة الرياضيات Math 102. الرجاء قراءة كل سؤال بعناية ثم ضع علامة √ داخل
المربع المقابل له إذا كنت تعتقد أن بإمكانك حله (لا تحاول حل السؤال)، و إذا كنت تعتقد أنه لا يمكنك حل السؤال فاترك المربع فارغاً هكذا ☐

Participated in: **Survey** 100 **Exit Quiz** 101

---

1. Do you know the following **Trigonometric Identities**, e.g.

a. $1+\tan^2\theta=\sec^2\theta$ 98

b. $\cos 2\theta=\cos^2\theta-\sin^2\theta$ 76

c. $\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}$ 60 59

d. $\csc\left(x-\frac{\pi}{2}\right)=-\sec x$ 33

2. Can you find *a*, *b* and *c* when $5-4x-2x^2$ is written in the **form** $a+b(x+c)^2$?

60

3. Can you do the **long division**, e.g., divide $5x^4+3x^2-4x-5$ by x^2-2x+4?

71

4. Can you sketch the **graphs** of following?

a. $y=|2x+3|+4$ 87

b. $y=x^2-5x+6$ 76 39

c. $y=x^3-4x$ 54

d. $y^2=x^3$ 52

e. $y=\ln(2x+1)-3$ 43

f. $y=e^{-(x+1)}-2$ 39

5. Can you check if the sequence $\left\{\frac{1-n}{2+n}\right\}_{n=1}^{\infty}$ is **increasing** or **decreasing**?

75

6. Can you find the following **limits**:

a. $\lim\limits_{n\to\infty}\left(1+\frac{2}{3n}\right)^{4n}$ 75

b. $\lim\limits_{n\to 0}n^2e^{-n}$. 53

c. $\lim\limits_{x\to\infty}\left(\sqrt{n}-\sqrt{n-1}\right)$ 75 52

d. $\lim\limits_{n\to\infty}\frac{2^n}{n!}$ 62

e. $\lim\limits_{n\to\infty}n\sin\frac{2}{5n}$? 30

7. Do you know all basic **derivative formulae**:

a. $\frac{d}{dx}\sec^{-1}(x+1)$ 60

b. $\frac{d}{dx}\log_5(x+8)$ 56

c. $\frac{d}{dx}3^x$ 84 67

d. $\frac{d}{dx}(x^2-3)^{-8}$ 86

8. Can you solve the **system of equations**?

a. $y=x^2$, $y=\sqrt{x}$ 92

b. $x^2+y^2=7$, $2x+y=-3$ 84 35

9. Do you know how to **Partition** [-3, 7] into 5 subintervals of equal length?

69

10. Can you solve **the definite integral** $\int_{-2}^{2}\sqrt{4-x^2}\,dx$ by using basic geometric formulae?

74 36

11. Do you know the difference between area and net signed area due to Definite Integrals?

54

12. Can you write the following finite sums in closed form:

$\sum_{k=1}^{n} k; \sum_{k=1}^{n} k^2; \sum_{k=1}^{n} k^3; \sum_{k=1}^{n} ar^{k-1};$. 64

13. Can you approximate an integral $\int_{-2}^{2} \sqrt{4-x^2}\,dx$ by using finite number of rectangles? 59

14. Do you remember **Basic anti-derivative** formulas e.g. for

a.	$(ax+b)^n$	75	
b.	$\tan x$	70	
c.	$\sec^2 x$	87	
d.	$\sec x$	67	
e.	$\coth x$	56	
f.	$\operatorname{csch} x$	56	
g.	$1/(1+x^2)$	82	79
h.	$1/\sqrt{1-x^2}$	80	
i.	$1/x\sqrt{x^2-1}$	78	

15. Do you know how to change the limits of integration in $\int_{-2}^{-1} x^2\sqrt{4-x^3}\,dx$ when you solve it by substitution method? 61

16. Do you know how to apply the Fundamental theorem of Calculus: $\frac{d}{dx}\int_{1}^{x}\sqrt{4+3t^3}\,dt$ 82

17. Do you know how to solve the following integral by substitution method?

a.	$\int x^2\sqrt{4-x^3}\,dx$	85	49
b.	$\int \sin x \cos^3 x\,dx$	78	65
c.	$\int \tan x \sec^4 x\,dx$	70	
d.	$\int \tan^3 x \sec^2 x\,dx$	84	51

18. Can you solve initial value problem: $\frac{dy}{dx} = \sin x + 2;\ y(\pi) = 4$. 57

19. Do you know how to find Local Quadratic or cubic Approximation of $f(x) = \ln x$ about a point $x = e$? 64

20. Do you know how to find formula for nth Derivative of the functions:

a.	$f(x) = e^{-3x}$	64	70
b.	$f(x) = \ln(2x+1)$	59	

21. Do you know the difference between Taylor Polynomial and Maclaurin Polynomial? 83

22. Can you write general term of a sequence: $\frac{2}{5}, \frac{-4}{8}, \frac{8}{11}, \frac{-16}{14}, \ldots$? 83 64

23. Do you know how to check if the sequence $\left\{\frac{1-n}{2+n}\right\}_{n=1}^{\infty}$ is increasing or decreasing? 70 54

24. Do you the meaning of eventually increasing sequence? 69

25. Can you find the limit of sequence $\{a_n\}_{n=1}^{\infty}$ where $a_n = \sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}$? 24

26. Do you know the difference between a sequence and a series? 86

27. Do you know the use of $\lim_{n\to\infty} S_n$ where S_n is the n^{th} partial Sum of an Infinite Series? 41

28. Do you know the two series for which the nth partial Sum can be written in Closed Form? 41

29. Do you know how to check Convergence or Divergence of improper integral $\int_{0}^{\infty}\frac{dx}{1+x^2}$? 64

30. Do you know how to apply following tests to check Convergence or Divergence of Infinite series?

a. Integral Test 78 40
b. Divergence Series Test 77
c. P-series test 90
d. Basic Comparison Test 79
e. Ratio Test 89 40
f. Root Test 80

31. Do you know how to check if the series $\sum_{k=2}^{\infty}\frac{(-1)^n}{n\ln n}$ is absolute convergent, conditional convergent or divergent? 55

32. What is the use of formula $|S-S_n|<a_{n+1}$ related to $\sum_{n=1}^{\infty}(-1)^n a_n, a_n>0$. 38

33. Do you remember the Power series of following functions:

a. $(1-x)^{-1}$ 54
b. e^x 63
c. $\sin x;\ \cos x$ 54

34. Do you know how to use standard series for finding Power Series Representation of

a. $1/\left(4+5x-4x^2\right)$ 29
b. $1/(1+x)^2$
(Term by Term Differentiation). 34
c. $\int_0^x e^{-t^3}dt$
(Term by Term Integration) 25

35. Can you find radius and interval of convergence of Power Series?
$\sum_{k=1}^{\infty}(-1)^n\frac{n^2}{n!}x^n$. 48

36. Do you remember the following formulas:

a. Volume of Washer 84 30
b. Volume of Shell 84 54
c. Surface Area of Frustum 69

37. Can you find the area of the region bounded by the graphs of $y=x^2-2, y=x$. 87

38. Do you know how to find Volume of a Solid which is generated by revolving the region bounded by graphs of $y=x^2-2, y=x$

a. About the line $y=3$ 67 58
b. $x=-4$ 71

39. Do you remember the formulas

i. For Arc Length 63 73
ii. For Area of Surface when a curve is revolved about *x*-axis or *y*-axis 59
iii. (a) and (b) For parametirc curves 38

40. Can you **integral** of

a. xe^x 68
b. $x\cos x$ 69 60
c. $e^x\cos x$ 55
d. $\sin^{-1}x$ 49

41. Do you remember the technique to simplify the expression $\sqrt{1+[f'(x)]^2}$ when

a. $f(x)=\frac{x^2}{8}-\ln x$ 43
b. $f(x)=\cosh x$ 46
c. $f(x)=\ln(\sin x)$ 38

****

Note: The figure in the white box represents the percentage of students who showed their awareness of the material included in that item. The figure in the **red box** against a **survey item** represents the average percentage score obtained by the students while giving the answer related to the item given in the Exit Quiz.

Code: 001 MATH 102-042 (Exit Quiz) Time: 1 hour *Appendix* 1(vi)

ID.___________ Section or Instructor ______________ Serial#:___

---

1. $\frac{d}{dx}4^x\Big]_{x=2}$

a. $8\ln 8$

b. $16\ln x$

c. $16\ln 4$

d. $4\ln 4$

e. 16

2. **Using basic geometric formula**, the value of integral $\int_{-2}^{0}\sqrt{4-x^2}\,dx$ =

a. -2π

b. 2π

c. $\pi/2$

d. 4π

e. π

3. The **anti-derivative** $\left(v^2+1\right)^{-1}$ is

a. $v/(v^2+1)+C$

b. $-2v(v^2+1)^{-2}+c$

c. $(v^2+1)^3/3+C$

d. $\tan^{-1}v+C$

e. $1/0$

4. $\int_0^1 9x^2\sqrt{1-x^3}\,dx$

a. 3

b. 2

c. 1

d. -2

e. -3

5. $\int 4\cos x\sin^3 x dx$

a. $\sin^4 x+C$

b. $4\sin^4 x/3+C$

c. $-\sin^4 x+C$

d. $\cos^4 x+C$

e. $-\cos^4 x+C$

6. The sequence $\left\{\frac{4+n}{5+n}\right\}_{n=1}^{\infty}$

a. is neither increaing nor decreaing

b. is decreaing

c. has limit $5/4$

d. is increaing

e. is eventually decreaing

7. Using the Integral Test for testing the series $\sum_{n=2}^{\infty}\frac{1}{100n\ln n}$, we find that the series is

a. Divergent because $\lim_{n\to\infty}\frac{1}{n\ln n}\neq 0$

b. Convergent because $\lim_{n\to\infty}\frac{1}{n\ln n}=0$

c. Divergent

d. Convergent because $\int_{2}^{1000}\frac{dx}{x\ln x}<\infty$

e. Convergent

8. The nth term of the sequence $\frac{2}{2},\frac{4}{8},\frac{8}{32},\frac{16}{128},\dots$ is given by

a. 2^{3-3n}

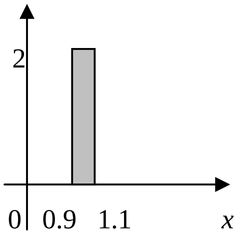
b. 2^{n-1}

c. 2^{2n-2}

d. 2^{2-2n}

e. 2^{1-n}

9. Volume of Cylindrical Shell when the Shaded Rectangle is revolved about y-axis is given by



a. $\pi[(1.1)^2-(0.9)^2]$

b. 0.8π

c. $2\pi[(1.1)-(0.9)]$

d. $2\pi[(1.1)^2-(0.9)^2]$

e. 0.4π

10. [Note: $\cos\pi=-1;\sin\pi=0$]

The value **integral** of $\int_{0}^{\pi}w\sin w\,dw$ is

a. π

b. 0

c. 2π

d. 4π

e. $3\pi/4$

11. The parabola $y = 3 + 2x + x^2$ has

a. vertex at (a, b) with $a + b = 5$
b. 2 real zeros
c. y-intercept at 6
d. vertex at (a, b) with $a + b = 1$
e. one absolute maxima

12. The 50th Derivative of the function e^{2x} is

a. $50e^{2x}$
b. $2^{50}e^{2x}$
c. e^{2x}
d. $50^2 e^{2x}$
e. $2x(2x-1)\cdots(2x-49)e^{2x}$

13. The system of equations $\begin{matrix} x^2 - y^2 = -1 \\ x + y = 1 \end{matrix}$

a. has a solution (a, b) with $a - 3b = 1$
b. has a solution (a, b) with $a - 3b = 2$
c. has a solution (a, b) with $a - 3b = 0$
d. has a solution (a, b) with $a - 3b = -2$
e. has two solutions

14. Volume of a Solid generated by revolving the region bounded by graphs of $y = 2\sqrt{x}, y = 0, x = 1$ about the line $y = 0$ is

a. $\pi / 4$
b. $2\pi / 4$
c. π
d. 4π
e. 2π

15. $\lim_{n\to\infty}\left(n - \sqrt{n^2 + 9n}\right)$

a. 0
b. ∞
c. $-9/2$
d. $-\infty$
e. -3

16. Using half-angle trigonometric identity, the integral $\int 8\cos^2(2w)dw$

a. $(\sin^3 2w)/6+C$

b. $4w+\sin 4w+C$

c. $(\sin^3 2w)/3+C$

d. $(\cos^3 2w)/6+C$

e. $4w+\cos 4w+C$

17. Using the Ratio Test for testing the series $\sum_{n=2}^{\infty}\frac{(2n-1)!}{(2n)!}$, we find that

a. The Ratio Test fails.

b. The series is Convergent by Ratio Test

c. The series is Divergent by Ratio Test

d. $\lim_{n\to\infty}\frac{(2n-1)!}{(2n)!}\neq 0$

e. $\frac{(2n-1)!}{(2n)!}=\frac{1}{2n-2}$

18. Arc Length of the curve $y=1/x$ between the point (1,1) and (2, ½) is given by

a. $\int_1^2 \frac{\sqrt{1+x^2}}{x^2}dx$

b. $\int_1^2 \frac{\sqrt{1+x^2}}{x}dx$

c. $\int_1^2 \frac{\sqrt{1+x^4}}{x^2}dx$

d. $\int_1^2 \sqrt{1+x^4}dx$

e. $\int_1^2 \frac{\sqrt{1+x}}{x}dx$

19. $\int 6\cot^5 x\csc^2 xdx$

a. $\cot^4 x\csc^3 x+C$

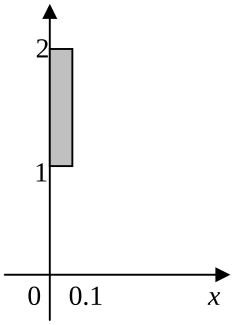
b. $2\cot^3 x/3+C$

c. $-\csc^4 x+C$

d. $-\cot^6 x+C$

e. $\cot^4 x+C$

20. Volume of Washer when the Shaded Rectangle is revolved about *x*-axis is given by



a. 0.4π

b. 0.1π

c. 3π

d. 0.6π

e. 0.3π

Appendix - II

Analysis of Survey I & Follow-up Quiz I (MATH 101-051)

Appendix 2(i)

الأسئلة الآتية متعلقة بمفاهيم سابقة تحتاجها في دراسة مادة الرياضيات Math 101. الرجاء قراءة كل سؤال بعناية ثم ضع علامة √ داخل
المربع المقابل له إذا كنت تعتقد أن بإمكانك حله (لا تحاول حل السؤال)، و إذا كنت تعتقد أنه لا يمكنك حل السؤال فاترك المربع فارغاً هكذا ☐

Participated in: **Survey** **559 Students** **Quiz** **383 Students**

---

1. Do you know the **Laws of Exponents,** e.g. $\dfrac{\left(2^3.2^{-2}\right)^4}{(3.2)^{-3}}$ 86% 61%

2. Do you know how to **simplify** rational expressions, e.g., $\dfrac{2-\dfrac{3}{x-1}}{x+\dfrac{2x}{3-x}}$ 86% 64%

3. Do you know the following **Trigonometric Identities**, e.g.

a. $1+\tan^2\theta=\sec^2\theta$ 92% 38%

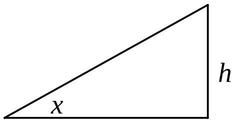
b. $\cos 2\theta=\cos^2\theta-\sin^2\theta$ 89% 69%

c. $\sin\dfrac{\theta}{2}=\pm\sqrt{\dfrac{1-\cos\theta}{2}}$ 86% 44%

d. $\csc\left(x-\dfrac{\pi}{2}\right)=-\sec x$ 59%

e. $\tan(a-b)=\dfrac{\tan a-\tan b}{1+\tan a\tan b}$ 87% 49%

4. Do you know how to use **Right Angled Triangle** to find **Trigonometric Ratios**, e.g. If $\sin x=h$ in the diagram, then



a. $\sec x={}^{1}\!/\!_{\sqrt{1-h^2}}$ 66% 71%

b. $\sin(2x)=2h\sqrt{1-h^2}$ 39%

c. $\cos\dfrac{x}{2}={}^{\sqrt{1+\sqrt{1-h^2}}}\!/\!_{2}$ 33%

5. Do you know if $x=3\sec\theta$, then $\sqrt{x^2-9}=3\tan x$ 26%

6. Can you find *a, b* and *c* when $5-4x-2x^2$ is written in the **form** $a+b(x+c)^2$? 56%

7. Can you do the **long division**, e.g., divide $5x^4+3x^2-4x-5$ by x^2-2x+4? 67%

8. Can you sketch the **graphs** of following functions?

a. $y=2x-3$ 90% 78%

b. $y=|x+2|$ 92% 44%

c. $y=|x^2-5x+6|$ 58%

d. $y=x^2$ 73% 47%

e. $x=y^2-5y+6$ 62%

f. $y=x^3-6$ 54%

g. $y=\sqrt{x-1}$ 58%

h. $y=\ln(x+1)$ 50%

i. $y=e^{-x}$ 54%

j. $y = \sec x$ 70% 51%

9. Can you **factorize**

a. x^3-64 89% 55%

b. $(x-2)^3$ 80% 56%

c. x^6-5x^3+6 75% 56%

d. $3x^3-x^2+3x-1$? 53%

10. Can you find the **solution** of $\sqrt{9x}=x+2$? 62%

11. A straight line passes through the points (0, – 4) and (–5,2). Can you find the slope of the line? 77% 70%

12. Do you know which one of the following is a function in *x*?

a. $y^2 = \frac{-x}{1+x}$

b. $y^3 = \frac{-x}{1+x}$

c. $y^2 + 1 = \frac{-x^2}{1+x^2}$ 20%

13. Can you find the Domain of the function:

$$y = \sqrt{\frac{-x}{1+x}}$$

65%

14. $t-2$ is a factor of the expression: $t^3 + 3t^2 - 12t + 4$. Can you find its second factor?

43%

15. Do you know the behavior of the graph of

$$1000x^{50} - 3x^{51}$$

when *x* increases indefinitely to the positive side,?

13%

16. Can you find the domain and range of the following functions:

a.	$\tan x$	81%	29%
b.	$\csc x$	80%	44%
c.	$\log(x-2)$	56%	

17. Can you find the inverse of

$f(x) = 2x + 3$? 69% 74%

18. For a given function $f(x) = \sqrt{x-1}$, can you find the domain and range of f^{-1}?

53%

19. Do you know the domain and range of an inverse trigonometric function?

54%

20. Do you know the difference between Radian measure and Degree of an angle?

47%

21. Can you solve the **system of equations**?

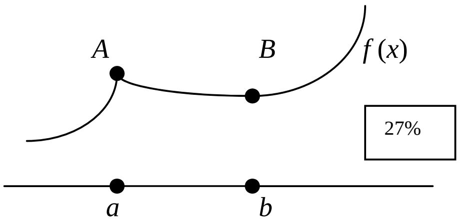
$$x^2 + y^2 = 7$$
$$2x + y = -3$$

62%

22. Do you know the **formulae** related to following **Geometrical figures**:

a.	Area of **Triangle**	69%	89%
b.	Area of **Trapezoid** of height *h*	28%	
c.	Volume of **Cylinder**	28%	
d.	Surface area of **Cylinder**	6%	
e.	Surface area of a **Cone?**	19%	
f.	Volume of a **Cone?**	3%	

23. Can you find the **distance between points** A and B given in the diagram:



27%

24. Consider the following function

$$y = \frac{x^2 - 2x + 1}{(x-1)(x-2)}.$$

Do you know how to find

a.	its vertical asymptote?	56%
b.	its horizontal asymptote?	49%
c.	The point where the graph has a hole?	46%

****

Note: The figure in the **red box** against a **survey item** represents the percentage of students who gave correct answer for the item when included in the **Quiz.**

Sec #:__________ **Quiz (MATH 101) [C 003]** **ID#__________** *Appendix* 2(ii)

One Hour

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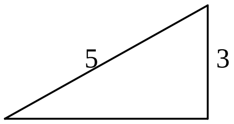
1. Area of the given **Triangle** is



   a. 15
   b. 8
   c. 6
   d. $\frac{15}{2}$.

2. When x^4+2x^2-4x+6 is divided by x^3-x+1, the remainder is given by

   a. $3x^2-3x+6$
   b. $3x^2-x-1$
   c. x^2-3x+6
   d. x^2-5x+6

3. If $\tan x=\frac{1}{h}$, then $\cos x =$

   a. $\frac{h}{\sqrt{h^2-1}}$
   b. $\frac{\sqrt{1+h^2}}{\sqrt{1-h^2}}$
   c. $\frac{h}{\sqrt{1+h^2}}$
   d. $\sqrt{1+h^2}$

4. The graph of $y=\sec x$ is

   a. is increasing in the interval $\left(0,\frac{\pi}{2}\right)$
   b. is undefined at $x=0$
   c. is increasing in the interval $\left(\frac{-\pi}{2},0\right)$
   d. is decreasing in the interval $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

5. $\dfrac{1-\dfrac{3}{x-1}}{4+\dfrac{2x}{2-x}} =$

a. $\dfrac{2-x}{x-1}$

b. $\dfrac{2-x}{3(x-1)}$

c. $\dfrac{x-2}{x-1}$

d. $\dfrac{x-2}{2(x-1)}$

6. The function $y = \tan x$ is

a. defined in the interval $\left(\dfrac{-3\pi}{2}, 0\right)$

b. a bounded function

c. not defined in the interval $(0, \pi)$

d. not defined in the interval $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$.

7. The inverse function of $f(x) = 3x + 2$ is given by

a. $f^{-1}(x) = \dfrac{1}{3}(x+2)$

b. $f^{-1}(x) = 3x - 2$

c. $f^{-1}(x) = \dfrac{1}{3}(x-2)$

d. $f^{-1}(y) = \dfrac{1}{3}(y+2)$

8. One of the factors of $x^4 - 5x^2 + 6$ is given by

a. $x - \sqrt{3}$

b. $x + 3$

c. $x + 2$

d. $x - 3$

9. The graph of $x = y^2 + 1$ is a parabola which

a. Opens on the left side of y-axis.

b. Has axis along the y-axis

c. Has the vertex at (1,0)

d. Has vertex at (0,1)

10. Using “Half-Angle Formula”, only one of the following holds

a. $\sin 200^\circ = \sqrt{\dfrac{1-\cos\ 400^\circ}{2}}$

b. $\sin 100^\circ = -\sqrt{\dfrac{1-\cos\ 200^\circ}{2}}$

c. $\sin 100^\circ = \sqrt{\dfrac{1+\cos\ 200^\circ}{2}}$

d. $\sin 200^\circ = -\sqrt{\dfrac{1-\cos\ 400^\circ}{2}}$

11. The range of the function $y = \csc x$ is

a. $(-\infty, 0) \cup (0, \infty)$

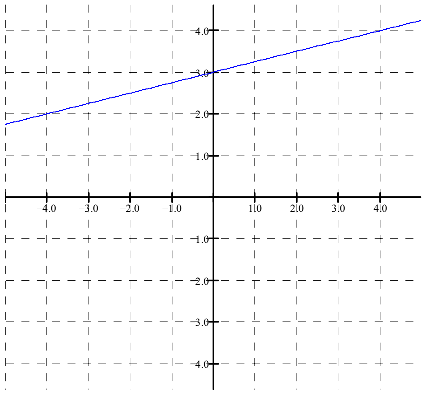
b. $(-1, 1)$

c. $(-\infty, \infty)$

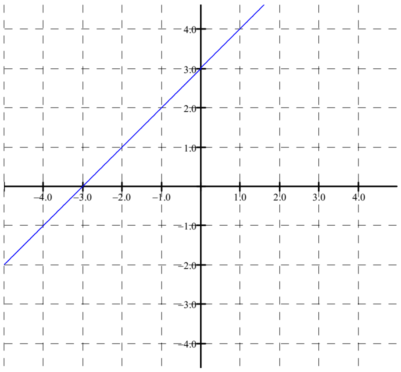
d. $(-\infty, -1] \cup [1, \infty)$

12. The graph of the equation $y = -2x + 3$ is:

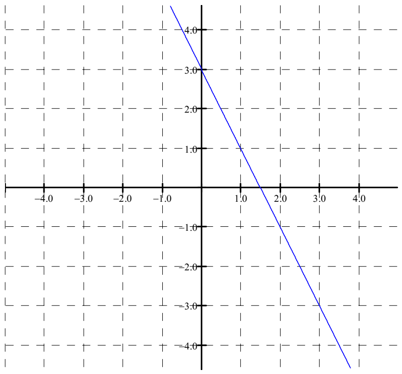
a



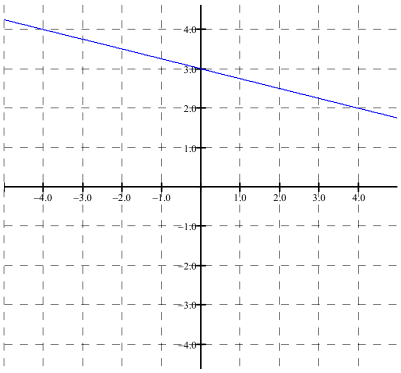
b



c



d



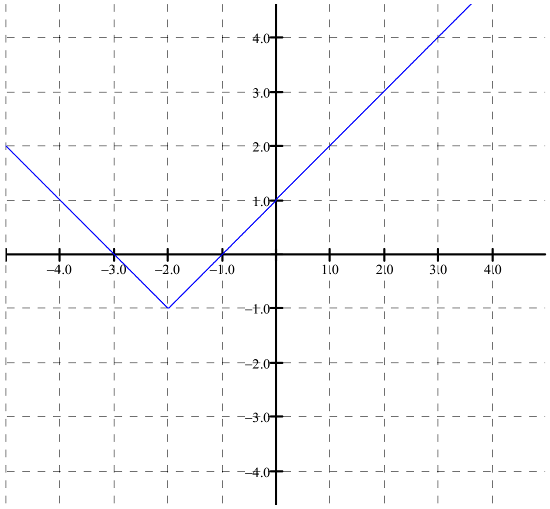
13. One factor of $x^3 - 64$ is:

a. $x^2 + 4x + 16$
b. $x^2 - 8x - 16$
c. $x^2 - 8x + 16$
d. $x^2 + 4x - 16$

14. The coefficient of x^2 in the product $(x-2)^3$ is:

a. -12
b. 6
c. 12
d. -6

15. The equation that represents the following graph is:



a. $|x+2|+1$
b. $|x+2|-1$
c. $|x-2|-1$
d. $|x-2|+1$

16. The slope of the line that passes through the points (0,7) and (2,3) is

a. 2
b. $-\frac{1}{5}$.
c. -2
d. 5

17. $\dfrac{\left(3^5.3^{-2}\right)^{\frac{-1}{2}}}{3^{-1}}$

a. $\dfrac{1}{\sqrt{3}}$

b. $3^{\frac{3}{2}}$

c. $\sqrt[3]{3}$

d. $\dfrac{1}{3}$

18. $1-\sec^2\theta =$

a. $\tan\boldsymbol{\theta}$

b. $-\dfrac{\sin^2\boldsymbol{\theta}}{\cos^2\boldsymbol{\theta}}$

c. $\tan^2\boldsymbol{\theta}$

d. $\tan^2\boldsymbol{\theta}-1$

19. $\tan\left(\dfrac{\pi}{4}-a\right)=$

a. $(\tan a-1)/(\tan a+1)$

b. $(-\tan a+1)/(\tan a+1)$

c. $\tan a/(\tan a+1)$

d. $\tan a/(\tan a+1)$

20. $\cos 2x =$

a. $1-2\cos^2 x$

b. $1-2\sin^2 x$

c. $2\sin x\cos x$

d. $2\sin^2 x-1$

Analysis of Survey II & Midterm Exam (MATH 101-052)

Appendix 2(iii)

الأسئلة الآتية متعلقة بمفاهيم سابقة تحتاجها في دراسة مادة الرياضيات **Math 101**. الرجاء قراءة كل سؤال بعناية ثم ضع علامة √ داخل المربع المقابل له إذا كنت تعتقد أن بإمكانك حله (لا تحاول حل السؤال)، و إذا كنت تعتقد أنه لا يمكنك حل السؤال فاترك المربع فارغاً هكذا ☐

Participated in: **Survey** 93 **Midterm Exam** 109

1. Do you know about

i. $\lim_{x\to a} f(x)=L$?	99	
ii. $\lim_{x\to a} f(x)=\infty$?	92	
iii. $\lim_{x\to -\infty} f(x)=L$?	88	
iv. one-sided limit?	91	96
v. relationship between one-sided and two sided limit?	80	
vi. Squeezing Theorem?	54	

2. Do you know how to find following limits:

a. $\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x}-\sqrt{x-1}\right)$	47	
b. $\lim_{x\to 3}\frac{x-3}{x^2-5x+6}$.	92	
c. $\lim_{x\to\infty}\frac{8x^2-3x-11}{5-3x+7x^2}$	88	
d. $\lim_{x\to -\infty}\tan^{-1}x$	42	
e. $\lim_{x\to 0}\frac{\sin 3x}{x}=$	84	83.1
f. $\lim_{x\to 3}\frac{x^3-27}{x-3}$	100	
g. $\lim_{x\to -\infty}(6x^2-5x^3-8)$	83	94.1

3. Do you know how to apply $(\varepsilon-\delta)$-definition of limit to show that $\lim_{x\to -1}(3x-1)=-4$?	62	81.5

4. Do you know

i. how to show a function $f(x)$ is continuous at a point a?	90	
ii. when is a rational function continuous at a point a?	57	
iii. under what condition can we write $\lim_{x\to a} f(g(x))=f(\lim_{x\to a} g(x))$?	38	
iv. when is a function continuous at an end point of interval $[a,b]$?	69	
v. about removable discontinuity?	47	44.4
vi. about slope of tangent line to the graph of a function f (x) at a point a?	73	
vii. equation of the tangent line to the graph of $y=\sin x$ at $\frac{\pi}{6}$?	61	71.7
viii. about instantaneous velocity?	81	
ix. about average and instantaneous rate of change?	77	75.8
x. the definition of derivative of a function?	78	70
xi. Left hand and right hand derivatives?	53	
xii. Function with infinite slope at a point?	32	
xiii. Function with corner point at a point?	51	
xiv. Higher derivatives?	87	
xv. Product rule for derivatives?	86	
xvi. Quotient rule for derivatives, e.g. $\frac{d}{dx}\left(\frac{x}{x+1}\right)$?	88	

xvii. Formulas for derivative of Trigonometric functions? 94

xviii. Chain rule for derivatives

e.g. $\frac{d}{dx}\sin(x^2-3)^{-8}$? 81 87.2

xix. the Related rates? 44 75.1

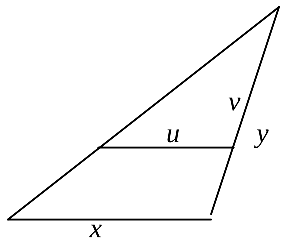
xx. Use of chain rule in Related rates? 47

5. Do you know how to find the derivatives of the following functions using the definition?
   - i. $f(x)=\sqrt{x}$ 88
   - ii. $f(x)=\sin x$ 84 45.3

6. Do you know about
   - i. Local linear approximation (L. L. A.) of a function at a point? 65 64.6
   - ii. Using L. L. A. to approximate numbers like $\sqrt{4.001}, \sin 31^\circ, (10.03)^8$? 49
   - iii. Differential of a function? 44
   - iv. Differential form of L. L. A.? 34
   - v. Approximation of error using differentials? 47 64.2
   - vi. Relative error? 66
   - vii. Percentage error? 68

7. Do you know the formula for finding
   - i. Area of triangle? 91
   - ii. Area of circle? 87
   - iii. Area of circular sector? 39
   - iv. Volume of sphere? 55
   - v. Surface area of sphere? 31
   - vi. Volume of cone? 49
   - vii. Surface area of cone? 26

viii. Relation between *x*, *y*, *u*, *v* in the following diagram (Similar Triangles)? 69



8. Can you sketch the **graphs** of following functions?

a. $y=2x-3$ 90

b. $y=|x+2|$ 87

c. $y=x^2$ 95

d. $y=x^2-5x+6$ 46

e. $y=x^3-6$ 54

f. $y=\sqrt{x-1}$ 53

9. Word problems

a. I **can solve** with some effort 55 61.1

If your answer for 9 (a) is no, go to question 9(b).

b. I **cannot** solve **<u>because of</u>** difficulty in

i. English Language 49

ii. Sketching the Diagram 26

iii. Geometrical Formulas 33

iv. Identifying the variables 24

v. finding the relationship in variables 47

King Fahd Univesity of Petroleum & Minerals
Department of Mathematical Sciences

MATH 101-051 (Midterm Exam)
(November 27, 2005)

Name:______________________________ ID:______________ Sec. #:_____ Serial #________

Time: 90 minutes

Max Points: 100

Show all necessary work

No calculator is allowed in the exam.

Points: ________/100

Part (I)
(Each question carries 5 pts)

Q.1. **Evaluate** $\lim\limits_{t\to 2^-}\dfrac{|2t-4|}{2-t}$

Q.2. **Evaluate** $\lim\limits_{y\to -\infty}\left(1+20y^3-12y^6+5000y^5\right)$.

Q.3. **Evaluate** $\lim\limits_{w\to 0}\dfrac{1-\cos w}{\sin w}$

Q.4. Only **using the idea of derivative**, find

$$\lim_{t\to 0}\frac{\cos(y+t)-\cos y}{t}$$

Part (II)

(Each question carries 7 pts)

Q.5. Find a value of $\delta > 0$ which satisfies **the** $\left(\varepsilon - \delta\right)$ **definition of limit** when

$$\lim_{u \to -1/2} \frac{4u^2 - 1}{2u + 1} = -2;\ \varepsilon = 0.5 .$$

Q.6. Find all **Removable Discontinuity(ies)** for the function $f(x) = \dfrac{x-2}{(x+1)(x^2 - 3x + 2)}$.

Q.7. Using the **definition of derivative**, find $g'(1)$ when $g(v) = \dfrac{1}{v^2}$.

Q.8. Find the **equation of tangent line** to the graph of $f(x) = \dfrac{x}{1+x}$ at $x = 1$.

Q.9. **Find** $\dfrac{d}{dx}\sqrt{\cos(\pi x^2 - 5)}$.

Q.10. A **particle** is moving along a straight line such that its **position** is given by $s(t) = 5t^2 - t + 5$.

a) Find the **average velocity** of the particle in the time interval [1,3].

b) The **instantaneous velocity** of the particle at t=2.

Q.11. Let l be the **length of the diagonal** of a Rectangle. Suppose that **Sides of the Rectangle** have lengths x and y. Assume that x and y are **changing with time** t.

a) **Draw** a Picture of the Rectangle **with the labels** x, y and l.

b) How are x, y and l **related**?

c) How are dx/dt, dy/dt and dl/dt **related**?

d) As x **increases** at a constant rate of ½ ft/s and y **decreases** at a constant rate of ¼ ft/s, how fast is the **size of Diagonal** changing when $x = 3$ ft and $y = 4$ ft.

e) Is the length of Diagonal **Increasing or Decreasing** at that instant? Give Reason.

Q.12. **(a)** Find the **Local Linear Approximation** of $\frac{1}{\sqrt{x+2}}$ **at** $x_0 = 2$.

(b) Use (a) to **approximate** $\frac{1}{\sqrt{4.01}}$

Q.13. The **side of a square** is measured with a possible **percentage error** of $\pm 1\%$. Use **differentials** to estimate the **percentage error in the area.**

Part (III) (8+9 pts)

Q.14. Water is pouring **at the rate of** 6ft^3/min in a **cylindrical tank** of **radius** 120ft. **How fast** is the **height of water** rising up in the cylinder? [**Note**: Volume of Cylinder = $\pi r^2 h$].
(Show complete work)

Q.15. **Find value(s)** of k so that $f(x)=\begin{cases}\dfrac{\sin 2k(x-1)}{7(x-1)}, & x\neq 1\\ k-1, & x=1\end{cases}$ is **continuous at** x = 3.

Analysis of Survey III & Exit Quiz (MATH 101-051)

Appendix 2(v)

الأسئلة الآتية متعلقة بمفاهيم سابقة تحتاجها في دراسة مادة الرياضيات Math 101. الرجاء قراءة كل سؤال بعناية ثم ضع علامة √ داخل المربع المقابل له إذا كنت تعتقد أن بإمكانك حله (لا تحاول حل السؤال)، و إذا كنت تعتقد أنه لا يمكنك حل السؤال فاترك المربع فارغاً هكذا ☐

Participated in: **Survey (E-Students)** 100 **Exit Quiz (E-Students)** 87

---

1. Can you find the following **limits**:

a.	$\lim_{x\to 2}\frac{2-x}{x^2-5x+6}$?	100	
b.	$\lim_{x\to 2^-}\frac{\lvert 2-x\rvert}{x^2-4}$?	90	43.7
c.	$\lim_{x\to 2^+}\left(\frac{\sqrt{x^2-1}-\sqrt{3}}{x-2}\right)$?	79	39.1
d.	$\lim_{x\to-\infty}\left(150-60x^2-2x^3\right)$?	97	
e.	$\lim_{x\to 0}\frac{\sin(\pi-2x)}{x}$?	56	

2. Using the concept of limit, can you find all horizontal and vertical asymptotes $f(x)=\frac{x^2-4}{x^2-5x+6}$? 72

3. Using the $(\varepsilon-\delta)-$ definition can you show that $\lim_{x\to-2}\frac{2x^2-8}{x+2}=-8$? 85 | 24.1

4. Can you **estimate** $\sin 1^\circ$ or $\sqrt[3]{26.09}$ **using Differentials**? 54 | 44.8

5. Can you find the **vaule(s) of *k*** for which

$$f(x)=\begin{cases} x+k^2 & x\le 0 \\ k+x & x>0 \end{cases}$$

a.	is **continuous**?	85	67.7
b.	is **only** left continuous?	59	

6. Can you find all the **removable discontinuity(ies)** of $f(x)=\frac{2-x}{x^2-5x+6}$? 62

7. Using **L'Hopital Rule** Can you evaluate?

a.	$\lim_{x\to\infty} x^2 e^{-x}$.	62	
b.	$\lim_{x\to\infty}\frac{2^x}{x^2}$	69	40.2
c.	$\lim_{x\to\infty} x\sin\frac{2}{5x}$?	59	

8. Can you evaluate the **Indeterminate Form**?

a.	$\lim_{x\to\infty}\left(1+\frac{2}{3x}\right)^{4x}$	44	
b.	$\lim_{x\to 0}\left(x^{-1}-\left(e^x-1\right)^{-1}\right)$	26	
c.	$\lim_{x\to\infty}\left(e^x+x\right)^{1/x}$	37	

9. Can you find the **derivatives** of following functions by **using definition**:

a.	$f(x)=x^2$	97	
b.	$f(x)=\sin x$	85	
c.	$f(x)=\log x$	64	

10. Do you know the relation between **derivative** of a function $f(x)$ **and**

a.	**Slope** of tangent line to $f(x)$	95	46
b.	**Velocity** of a particle having position function $f(x)$	67	
c.	**Instantaneous rate of change** of $f(x)$?	67	56.3

11. Do you know how to **differentiate**

a.	$\frac{d}{dx}\sec^{-1}(x+1)$?	79	
b.	$\frac{d}{dx}\log_5(x+8)$?	79	34.5
c.	$\frac{d}{dx}3^x$?	79	
d.	$\frac{d}{dx}(x^2-3)^{-8}$?	87	66.7

Please see the other side

12. Do you understand the

a. **Sandwich** Theorem? 62

b. **Intermediate Value** Theorem? 41

c. **Product Rule** for derivatives? 95

d. **Quotient Rule** for derivatives? 95

e. **Chain Rule** for derivatives? 95 74.7

f. Derivative of **Inverse of a function**? 62

g. **Related** Rates? 64

h. **Local Linear** Approximation? 67

i. **Differential** of a function? 62

j. **Estimation of Error** using Differentials? 54

13. Can you solve **Word Problems** based on **Related Rates**? 59

14. Do you know how to find **higher derivatives** of a function? 79 74.7

15. Can you find *dy/dx* for $xy^2+\sin(xy)=5$ (***Implicit Differentiation***)? 62

16. Do you know how to find

a. **Critical points** of $f(x)$? 82

b. The intervals in which *f*(*x*) is **increasing or decreasing**? 90 82.8

c. The **points of Inflection** of $f(x)$? 85 50.8

d. The intervals in which $f(x)$ is **Concave Upward or Downward**? 85

17. Do you understand the geometrical meaning of the **order of a zero** of polynomial? 44

18. Can you check if **at a critical point** graph of $f(x)$ has

a. **Vertical Tangent** Line? 59

b. **Cusp?** 49

19. Do you know the use of **1st and 2nd Derivative Tests** to check Critical points for Relative Extrema ? 67

20. Do you know when does **2nd Derivative Test fails** while checking Relative Extrema? 62

21. Do you know how to find **for a Rational Function**

a. **Oblique** Asymptote? 28 13.8

b. **Curvilinear** Asymptote? 28

22. Do you know how to find for $f(x)$ **Absolute Maxima/Minima**

a. on **Closed Interval** ? 69

b. on **Open Interval**? 67

23. Do you know the **difference** between **Absolute Extrema** and **Relative Extrema** of $f(x)$? 54 57.5

24. Do you know how to solve **Word Problems** based on **maxima/Minima**? 33

25. Do you know the following **Trigonometric Identities**, e.g.

a. $1+\tan^2\theta=\sec^2\theta$ 74

b. $\cos 2\theta=\cos^2\theta-\sin^2\theta$ 69

c. $\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}$ 46

d. $\csc\left(x-\frac{\pi}{2}\right)=-\sec x$ 51

26. Do you know the following **formulas**?

a. $(a-b)^3=a^3-3a^2b+3ab^2-b^3$ 69

b. $a^3-b^3=(a-b)(a^2+ab+b^2)$ 74

Please see the other side.

Code: 003 **MATH 101-051 (Exit Quiz)** **Time: 1 hour** *Appendix* 2(vi)

Note: *The matching Survey Question is mentioned against each of the Exit Question*

# of Respondents from Sec 4, 8, 12, 17 = 87 **# of Respondents from Other Sections = 141**

---

1. $\lim_{u \to 3^-} \frac{|3-u|}{u^2-9} =$

a. $-\infty$

b. $1/3$

c. $-1/6$ *Survey Question: 1b*

d. ∞

e. $1/6$

2. $\lim_{x \to 1^-} \frac{1-\sqrt{2x-1}}{1-x} =$

a. 1

b. 2

c. 0 *Survey Question: 1c*

d. ∞

e. $-\infty$

3. The **sum** of all values of ***k*** which make

$$f(x)=\begin{cases} kx+4, & x \le 1 \\ k^2x^3+k, & x>1 \end{cases}$$

continuous for all real numbers is

a. 2

b. -2

c. 4 *Survey Question: 5*

d. 0

e. 1

4. $\lim_{w \to \infty} \frac{2^w}{10^4 w^2}$

a. $1/10^4$

b. ∞

c. 0 *Survey Question: 7b*

d. 10^4

e. none of the above

5. The **equation of the tangent line** to the graph of $f(x)=x^2+2x$ when $x = 2$ **contains the point**

a. $(-1,2)$

b. $(0,1)$

c. $(1,2)$ *Survey Question: 10a*

d. $(4,1)$

e. $(2,-1)$

6. When $\varepsilon=0.36$, the value of δ that satisfies the $(\varepsilon-\delta)$ **-definition** for $\lim_{x\to 3}(9-4x)=-3$ is given by

a. -0.12

b. -0.09

c. -0.36 *Survey Question: 3*

d. 0.04

e. 0.09

7. The **approximation** of $\sqrt{1.1}$ by **using differentials** and the function $f(x)=\sqrt{x}$, is given by

a. 1.01

b. 2.3/2

c. $3.2/3$ *Survey Question: 4*

d. $2.1/2$

e. $2.001/2$

8. $\frac{d}{dx}\left[\log_{100}(x+4)\right]_{x=6}$ =

a. $\ln 10/\ln 100$

b. $1/(20\ln 10)$

c. $\ln 100/10$ *Survey Question: 11b*

d. $100/\ln 10$

e. $1/\left(2\ln 100\right)$

9. $\frac{d}{dx}\tan^2(x^2) =$

a. $\frac{2\sin(x^2)}{\cos(x^2)}$

b. $\frac{2\sin(x^2)}{\cos^3(x^2)}$

c. $2x\tan(x^2)$ *Survey Question: 12e*

d. $\frac{4x\sin(x^2)}{\cos^3(x^2)}$

e. $\tan(x^2)\sec^2(x^2)$

10. If ***a*** and ***b*** are respectively **absolute maxima** and **absolute minima** of the function $f(x) = 4(\sin x) - 3$ in $[0, 2\pi]$ then ***ab*** =

a. -7

b. -3

c. 0 *Survey Question: 23*

d. 9

e. 3

11. The function $f(x) = \frac{x^3 - x^2}{x^4 - 1}$ has only

a. One Horizontal Asymptote
and Two Vertical Asymptote

b. One Horizontal Asymptote
and One Vertical Asymptote

c. Two Vertical Asymptote

d. One Horizontal Asymptote

e. One Vertical Asymptote

Survey Question: 21

12. $\frac{d^2}{dx^2}\left(\frac{x}{x+1}\right)_{x=2} =$

a. $-2/9$

b. $1/9$

c. $-2/27$ *Survey Question: 14*

d. $2/3$

e. $-2/18$

13. If the **first derivative** of a function *f* is given by $x(x-1)^2(x+3)$, then f is

a. increasing in $(-\infty,-3)\cup(-3,1)$

b. decreasing in $(-3,0)\cup(1,\infty)$

c. decreasing in $(-\infty,0)\cup(1,\infty)$

d. increasing in $(-\infty,1)$

e. increasing in $(-\infty,-3)\cup(0,\infty)$

Survey Question: 16b

14. $\frac{d}{dx}\left[\frac{1}{\sqrt{(x^2-3)^3}}\right]_{x=2} =$

a. $-3/2$

b. -6

c. $-2/3$

Survey Question: 11d

d. -3

e. $-1/6$

15. If **Second Derivative** of a function *f* is given by $x^2(x^2-1)(x+2)^2$, then f has **point of inflection** at (x, y) **only when**

a. $x=0, x=-1, x=1, x=-2$

b. $x=-1, x=1, x=-2$

c. $x=0, x=-2$

d. $x=0, x=-1, x=1$

e. $x=1,-1$

Survey Question: 16 c

16. The **instantaneous rate of change** in $g(x)=\frac{x^2}{\pi}+\sin x$ at $x=\frac{\pi}{2}$ is equal to

a. $\frac{\pi}{4}+1$

b. 2

c. 1

Survey Question: 10c

d. π

e. $\pi+1$

Appendix - III

How to Use Mathematica to solve Some Problems of Mathematics

Appendix 3(i)

1. Introduction

With the help of Mathematica, we can do

i. Arithmatic and Algebraic Manipulations,

ii. Solve Different Types of Equations,

iii. Sketch Graphs of Functions and Equations.

2. How to Start

i. Click on the icon “Mathematica”.

ii. Click on the new file.

iii. Type your operation

iv. Press the keys: Shift & Enter

(*We shall call these two keys*: **[SE] keys**)

v. You will find the Answer in the Next Line.

3. Basic Algebra Operations

Addition: + Subtration: **–** Square root: **sqrt**

Division: / Power: ^ Multiplication: *

(Examples)

Example 1. Type: 5+3 **outside** the **Existing Cell**

i. Press the [SE] keys.

ii. You will get on the screen:

In [1]:= *5+3*

Out [1]= *8*

Example 2. Type: – 7 ^3 **outside Existing Cell**

i. Press the [SE] keys.

ii. You will get on the screen:

In [2]:= – 7 ^3

Out [2]= *– 343*

Example 3. Type: **4 + ¼** outside Existing Cell

i. Press the [SE] keys.

ii. You will get on the screen:

In [3]:= 4 + ¼

Out [3]= $\frac{17}{4}$

Example 4. Type: **4. + ¼**

i. Press the [SE] keys.

ii. You will get on the screen:

In [4]:= 4.+1/4

Out [4]= 4.25

[***Check the difference between Examples 3 & 4***]

Example 5. Type: **2 ^ (1/3)**

i. Press the [SE] keys.

ii. You will get on the screen:

In [5]:= 2 ^ (1/3)

Out [5]= $2^{1/3}$

Example 6. Type: **N[2 ^ (1/3)]**

i. Press the [SE] keys.

ii. You will get on the screen:

In [6]:= N[2 ^ (1/3)]

Out [6]= 1.25992

Example 7. Type: **2 ^ (1/3) //N**

i. Press the [SE] keys.

ii. You will get on the screen:

In [7]:= **2 ^ (1/3) //N**

Out [7]= 1.25992

[Check the difference among Examples 5, 6 & 7]

4. Built in Functions

For	***Type***	**For**	***Type***
|– 2 |	**Abs[– 2]**	e^2	**Exp[2]**
sin(5 radian)	**Sin[5]**	$\sin^{-1}$(.3)	**ArcSin[.3]**
sin(5 Degrees)	**Sin[5 Degree]**	sinh(5)	**Sinh[5]**
Natural Log of 5	**Log[5]**	Log of 5 to base 3	**Log[3,5]**

Try the following Exercises

a. Find cos(30) and cos(30^o). Use the methods of Examples 5 & 7. Explain the answers.

b. Find ln(7.8), log(15), $\log_4$(9), e^8, $\tan^2$(12)

c. Evaluate: 5-2(8^2-60)/4 [Ans: 3]

d. Evaluate: $\sqrt{9}-\frac{1}{3}+\frac{4(18-4^3)}{8}$

e. Evaluate: 2$\sin^{-1}$(1/3) – $\cosh^2$(4/5).

5. Manipulating Algebraic Expressions

Example 1: Find the value of $6-3x^5$ for x=3.

Solution: Type: $6-3*x\wedge 5/.x\to 3$

i. Press the [SE] keys. You will get:

In [6]:= $6-3*x\wedge 5/.x\to 3$

Out [6]= –723

Example 2: Find the value of $\sqrt{x^2+y^2}$ when y=x+1 and x=3.

Solution: Type: $x=3; y=x+1;$ sqrt[x^2+y^2]

Press the [SE] keys.You will get:

In [6]:= $x=3; y=x+1;$ sqrt[x^2+y^2]

Out [6]= 5

Example 3: Expand $(x+y)^5$.

Solution: Type: Clear[x,y]; Expand[$(x+y)\wedge 5$]

Press the [SE] keys. You get

In [6]:= Clear[x,y]; Expand[$(x+y)\wedge 5$]

Out [6]= $x^5+5xy^4+10x^3y^2+10x^2y^3+5xy^4+y^5$

6. Solving Equations Numerically

Example 1: **(**Eq. of one variable**)**

Solve the equation $x^2 + x = 2$ in *x*.

<u>Solution</u>: Type: Solve**[***x*^2+*x*==2, *x***]**

Press the [SE] keys.You will get:

In [1]:= Solve**[***x*^2+*x*==2, *x***]**

Out [1]= $\{\{x \to -2\}, \{x \to 1\}\}$

Example 2: **(**Eq. of two variables**)**

Solve the equation $x^2 - 4 = 0, y^2 = x^2$ in *x* & *y*.

<u>Solution</u>: Type:

Solve**[{***x*^2–4==0, *x*^2= *y*^2},{*x*,*y*}**]**

Press the [SE] keys. You will get:

In [2]:=

Solve**[{***x*^2–4==0, *x*^2= *y*^2},{*x*,*y*}**]**

Out [2]=

$\{\{x \to -2, y \to -2\}, \{x \to -2, y \to 2\}, \{x \to 2, y \to -2\}, \{x \to 2, y \to 2\}\}$

Example 3: **(**Complicated Equation**)**

Solve the equation $\ln\left(x + \sqrt{1 + x^2}\right) == 2$ in *x*.

<u>Solution</u>: Type:

Solve**[**Log[*x*+Sqrt[1+*x*^2]]==2, *x***]**

Press the [SE] keys. You will get:

In [3]:=

Solve**[**Log[*x*+Sqrt[1+*x*^2]]==2, *x***]**

Out [3]= $\left\{\left\{x \to \frac{1}{2} e^{-2}\left(-1 + e^4\right)\right\}\right\}$.

7. Solving Equations Symbolically

Example 1: **(**Eq. of two variables**)**

Solve the equation $x^2 - k^2 = 0, y^2 = x^2$ in *x*, *y*.

<u>Solution</u>: Type:

Solve**[{***x*^2– *k*^2==0, *x*^2= *y*^2},{*x*,*y*}**]**

Press the [SE] keys. You will get:

In [2]:=

Solve**[**{*x*^2– *k*^2==0, *x*^2= *y*^2},{*x*,*y*}**]**

Out [2]=

$\{\{x \to -k, y \to -k\}, \{x \to -k, y \to k\}, \{x \to k, y \to -k\}, \{x \to k, y \to k\}\}$

Example 2: **(**Complicated Equation**)**

Solve the equation $\ln\left(x + \sqrt{a + x^2}\right) == b$ in *x*.

<u>Solution</u>: Type:

Solve**[**Log[*x*+Sqrt[*a*+*x*^2]]==*b*, *x***]**

Press the [SE] keys. You will get:

In [3]:=

Solve**[**Log[*x*+Sqrt[*a*+*x*^2]]==*b*, *x***]**

Out [3]= $\left\{\left\{x \to \frac{1}{2} e^{-b}\left(-a + e^{2b}\right)\right\}\right\}$.

8. Numerical Solutions of Equation(s)

Example 1: Find the roots of the equation:

$$x^3 + x + 1 = 0.$$

<u>Solution</u>: Type:

NSolve**[***x*^3+*x*+1==0, *x***]**

Press the [SE] keys. You will get:

In [1]:= NSolve**[***x*^3+*x*+1==0, *x***]**

Out [2]=

$$\left\{\begin{matrix}\{x \to -0.682328\}, & \\ \{x \to 0.341164 - 1.16154i\}, \{x \to 0.341164 + 1.16154i\}\end{matrix}\right\}.$$

Example 2: Find solution of the system of equations:

$$x + y = 2, x - 3y + z = 3, x - y + z = 0.$$

<u>Solution</u>: Type:

$\text{NSolve}\left[\{x + y == 2, x - 3y + z == 3, x - y + z == 0\}, \{x, y, z\}\right]$

Press the [SE] keys. You will get:

In[2]:=

$\text{NSolve}\left[\{x + y == 2, x - 3y + z == 3, x - y + z == 0\}, \{x, y, z\}\right]$

Out [2]= $\{\{x \to 3.5, y \to -1.5, z \to -5.\}\}$.

Example 3: Find approximate solution of the equation:

$$3\cos x = \ln x$$

starting the approximation at *x*=1

<u>Solution</u>: Type:

$$\text{FindRoots}\left[3\cos[x] == \text{Log}[x], \{x, 1\}\right]$$

Press the [SE] keys. You will get:

In [3]:= $\text{FindRoots}\left[3\cos[x] == \text{Log}[x], \{x, 1\}\right]$

Out [3]= $\{x \to 1.44726\}$.

9. Sketching 2-D Graphs

Example 1: Draw graph of *y* = sin *x* when $0 \le x \le \pi$.

<u>Solution</u>: Type Plot[cos[*x*],{*x*, 0, *π*}]

Press the [SE] keys. You will get:

In [3]:= Plot[cos[*x*],{*x*, 0, *π*}]

Out [3]= ***You find the graph of*** cos *x* ***when*** $0 \le x \le \pi$.

Example 2: Draw graph of $f(x) = \begin{cases} x^3 - 1, & x \ge 0 \\ x^2, & x < 0. \end{cases}$

when $-4 \le x \le 5$.

<u>Solution</u>: Type

f[x_]:= x^3 – 1 /; x≥0

f[x_]:= x^2/; x < 0.

Press the [SE] keys. You will get:

In [3]:= f[x_]:= x^3 – 1 /; x≥0

f[x_]:= x^2/; x < 0.

Out [3]= **……….**

Review Material at the Start Calculus II (Deficiency) *Appendix* 3(ii)

1. The Laws of Exponents:

i. If a, b are reals and $r,\ s$ are intgers then

- $a^r a^s = a^{r+s}$
- $\left(a^r\right)^s = a^{rs}$
- $\left(ab\right)^r = a^r b^r$
- $\dfrac{a^r}{a^s} = a^{r-s}$

$$\frac{\left(2^{23}.2^{-3}.3^{-10}\right)^{-1/5}}{\left(3.2\right)^{-5}} \qquad \left[Ans.\ \frac{2}{27}\right]$$

Method: Simplify numerator and denominator separately. [*Patience is required for simplification*]

Remember:

- When is $\sqrt[n]{a}$ defined?

  If $a < 0$ then n must be ODD.

- When x is a variable,

  $\sqrt{x^2} = |x|$.

2. Simplifications

i. Simplify complex expression, e.g.,

$$\frac{\dfrac{5}{x+1}+\dfrac{2x}{x+3}}{\dfrac{x}{x+1}+\dfrac{7}{x+3}} \qquad \left[Ans.\ \frac{2x^2+7x+15}{x^2+10x+7}\right]$$

Method: Simplify numerator and denominator separately. [*Patience is required for simplification*]

ii. Simplify the following:

a. $\dfrac{9x^2-4}{3x^2-5x+2}\cdot\dfrac{9x^4-6x^3+4x^2}{27x^4+8x}$ $\left[Ans.\ \dfrac{x}{x-1}\right]$

b. $\dfrac{5x^2+12x+4}{x^4-16}\div\dfrac{25x^2+20x+4}{x^2+2x}$

$$\left[Ans.\ \frac{x}{\left(x^2+4\right)\left(5x+2\right)}\right]$$

Method: Factorize numerators and denominators and use the formulas:

- $a^3+b^3=(a+b)(a^2-ab+b^2)$
- $a^3-b^3=(a-b)(a^2+ab+b^2)$

iii. Simplify the following:

a. $\dfrac{2x+1}{x^2+4x+4}-\dfrac{6x}{x^2-4}+\dfrac{3}{x-2}$ $\left[Ans.\ -\dfrac{x+5}{(x+2)^2}\right]$

Method: Factorize the denominators and find the LCM.

b. $\dfrac{2\left(4x^2+9\right)^{1/2}-(2x+3)\left(\frac{1}{2}\right)\left(4x^2+9\right)^{-1/2}(8x)}{\left(\left(4x^2+9\right)^{1/2}\right)^2}$

$$\left[Ans.\ \frac{6(3-2x)}{\left(4x^2+9\right)^{3/2}}\right]$$

Method: Take out common factors in the numerator.

iv. Rationalize the Denominators

- $\dfrac{81x^2-16y^2}{3\sqrt{x}-2\sqrt{y}}$ $\left[Ans.\ \left(9x+4y\right)\left(3\sqrt{x}+2\sqrt{y}\right)\right]$

Method: Use of formula:

$$\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=a-b$$

- $\dfrac{1}{\sqrt[3]{u}-\sqrt[3]{v}}$ $\left[Ans.\ \dfrac{\sqrt[3]{u^2}+\sqrt[3]{uv}+\sqrt[3]{v^2}}{u-v}\right]$

Method: Use of formula:

$$\left(\sqrt[3]{a}-\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}\right)=a-b$$

3. Review Trigonometric Identities

a. $1+\tan^2\theta=\sec^2\theta$

b. $\cos 2\theta=\cos^2\theta-\sin^2\theta$

c. $\sin\dfrac{\theta}{2}=\pm\sqrt{\dfrac{1-\cos\theta}{2}}$

d. $\csc\left(x-\dfrac{\pi}{2}\right)=-\sec x$

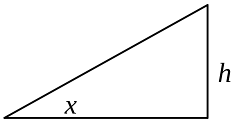
e. $\cos(\boldsymbol{\pi}-\boldsymbol{\theta})=-\cos\boldsymbol{\theta}$

f. $\tan(a-b)=\dfrac{\tan a-\tan b}{1+\tan a\tan b}$

[See similar other Identities in your Textbook]

4. Use of Right Angled Triangle to find Trigonometric Ratios

e.g. (i) If $\sin x = h$ in the diagram, then



a. $\sec x = {}^{1}\!/\!_{\sqrt{1-h^2}}$

b. $\sin(2x)=2h\sqrt{1-h^2}$

c. $\cos\dfrac{x}{2}={}^{\sqrt{1+\sqrt{1-h^2}}}\!/\!_{2}$

(ii) Use Right Angled Triangle to check:

If $x=3\sec\theta$, then

$\sqrt{x^2-9}=3\tan x$.

Read Page 44-48 & 64-74 of the Text for the following Topics

5. Sketch the Graph of Basic Functions

e.g. i. $y=\sqrt{x};\ y=x^2;\ x=y^2;\ y=x^3;\ y=|x|$

ii. $y=e^x;\ y=\ln x;$

iii. $y=2^x;\ y=\left(\dfrac{1}{2}\right)^x;$

iv. $y=\log_2 x;\ y=\log_{\frac{1}{2}} x$

v. All trigonometric Functions

6. Symmetry about Axes and the Origin

Recall the Tests for Symmetries.

7. Vertical & Horizontal Translation of Simple Graphs

e.g. Sketch the graphs of

$y=(x-1)^3+2;\ y=\sqrt{x+2}-1;\ y=\sin(x+1)-2$

$y=e^{x+1}-2;\ y=\ln(x+1)-2;$

8. Writing Quadratic Function in Standard Form

Find *a*, *b* and *c* when $y=5-4x-2x^2$ is written in the **form** $y=a+b(x+c)^2$? Then sketch the graph of the function

(Check: Vertex, *x* & *y* intercepts, parabola opens upwards or downwards) See page:

9. Long Division

Divide $5x^4+3x^2-4x-5$ by x^2-2x+4 and find Divisor; Quotient; Remainder.

10. Area of Geometrical Figures

Triangle, Trapezoid, Circle; Sector.

11. Factorize

$x^3-64;\ x^6-5x^3+6;\ 3x^3-x^2+3x-1$

12. Chain Rule for differentiation

Rule: Differentiate Extreme outer function, the next outer function, and then next….

e.g. find the derivative of

- $y=\left(\sin^3(x^2-3)+e^{\cos x^3}\right)^4$
- $y=\ln\left(\sin\sqrt[5]{x^3+5}\right)^6$.

[Identify the outer functions in order.]

MATH 102: Recitation Class

Appendix 3(iii)

Instructions:

I. Before going to Recitation Class, the students should

1. copy each problem on a separate sheet,
2. read the statement of each problem before coming to the class.

II. During the Recitation Class, the students should

1. sit in the assigned group,
2. sign attendance sheet,
3. discuss problems with each other in the group,
4. solve all the assigned problems,
5. be prepared to present solution of any question on blackboard,
6. be prepared to write a short quiz at the end of class.

Problems for Week 11

Q1. Using the power series of e^x and cos *x*, find the Power series representations of the function $f(x) = \cos^2(5x^3)$ **.** Also state its Radius of Convergence:

Q2. Using the power series of 1/(1-*x*), find the Power series representations of $f(x) = \frac{1}{9x^2 - 18x + 13}$. Also state its Radius of Convergence:

Q3. Using appropriate power series, approximate the integral $\int_0^{\frac{1}{2}} \frac{x}{64 + x^6}\,dx$ up to 3 decimal places.

Q4. Find the 1st 4 terms of the power series of $\csc x$ **(Hint: Use division of power series)**

Q5. Find the points of intersection of the following pair of functions and sketch the region in xy-plane:

$$f(x) = x^2,\ g(x) = 2x - x^2$$

Q6. **Sketch** the region enclosed by the curves. **Decide** whether to integrate with respect to *x* or *y*. **Draw** a typical approximating Rectangle and label its height and width. Then **find** the area of the region.

$$y = \sin x,\ y = e^x; x = 0, y = x^4$$

Q7. Find the number *a* such that the area of the region enclosed by the parabolas $y = x^2 - a^2$ and $y = a^2 - x^2$ is 576.

Review Material at the Start Calculus I (Deficiency) *Appendix* 3(iv)

1. The Laws of Exponents

i. Remember for $\sqrt[n]{a} = a^{1/n}$: If $a < 0$ then n must be Odd.

ii. If a, b are reals and $r,\ s$ are Rational Numbers then

- $a^r a^s = a^{r+s}$
- $\left(a^r\right)^s = a^{rs}$
- $\left(ab\right)^r = a^r b^r$
- $\dfrac{a^r}{a^s} = a^{r-s}$

Simplify $\dfrac{\left(2^{23}.2^{-3}.3^{-10}\right)^{-1/5}}{(3.2)^{-5}}$ $\left[Ans.\ \dfrac{2}{27}\right]$

Method: Simplify numerator and denominator separately. [*Patience is required for simplification*]

2. Importance of $\sqrt{x^2} = |x|$.

Find the horizontal asymptote(s) (H.A.) of

$$f(x) = \frac{2x^5 - 3x + 1}{\sqrt{5 + 9x^{10} - x^4}} \quad \left[Ans: y = \frac{2}{3};\ y = \frac{-2}{3}\right]$$

Note: For H.A., you have to check both limits:
$x \to \infty$ and $x \to -\infty$

3. Simplifications

i. Simplify complex expression

$$\frac{\dfrac{5}{x+1} + \dfrac{2x}{x+3}}{\dfrac{x}{x+1} + \dfrac{7}{x+3}} \quad \left[Ans.\ \frac{2x^2 + 7x + 15}{x^2 + 10x + 7}\right]$$

Method: Simplify numerator and denominator separately. [*Patience is required for simplification*]

ii. Simplify the following

a. $\dfrac{9x^2 - 4}{3x^2 - 5x + 2} \cdot \dfrac{9x^4 - 6x^3 + 4x^2}{27x^4 + 8x}$ $\left[Ans.\ \dfrac{x}{x-1}\right]$

b. $\dfrac{5x^2 + 12x + 4}{x^4 - 16} \div \dfrac{25x^2 + 20x + 4}{x^2 + 2x}$

$\left[Ans.\ \dfrac{x}{\left(x^2+4\right)\left(5x+2\right)}\right]$

Method: Factorize numerators and denominators and use the formulas (must remember these formulas):
[Also, see (9) for factorization]

- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

iii. Simplify the following:

a. $\dfrac{2x+1}{x^2 + 4x + 4} - \dfrac{6x}{x^2 - 4} + \dfrac{3}{x-2}$ $\left[Ans.\ -\dfrac{x+5}{(x+2)^2}\right]$

[**Method:** *Factorize the denominators and find the LCM.*]

b. $\dfrac{2\left(4x^2+9\right)^{1/2} - (2x+3)\left(\frac{1}{2}\right)\left(4x^2+9\right)^{-1/2}(8x)}{\left(\left(4x^2+9\right)^{1/2}\right)^2}$

$\left[Ans.\ \dfrac{6(3-2x)}{\left(4x^2+9\right)^{3/2}}\right]$

[**Method:** *Take out common factors in the numerator.*]

iv. Rationalize the Denominators

- $\dfrac{81x^2 - 16y^2}{3\sqrt{x} - 2\sqrt{y}}$ $\left[Ans.\ (9x+4y)\left(3\sqrt{x} + 2\sqrt{y}\right)\right]$

[Use of formula: $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b$]

- $\dfrac{1}{\sqrt[3]{u} - \sqrt[3]{v}}$ $\left[Ans.\ \dfrac{\sqrt[3]{u^2} + \sqrt[3]{uv} + \sqrt[3]{v^2}}{u - v}\right]$

[Use :$\left(\sqrt[3]{a} - \sqrt[3]{b}\right)\left(\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2}\right) = a - b$]

4. Completion of Square

Find a, b and c if $5 - 4x - 2x^2$ is written in the **form** of $a + b(x+c)^2$?

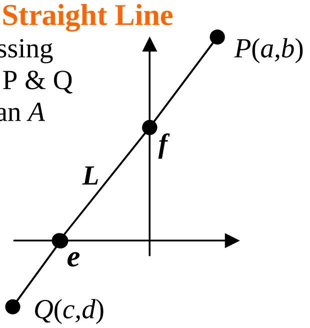
Sol. (Re-write the expression in descending order)

$$5 - 4x - 2x^2 = -2x^2 - 4x + 5$$
$$= -2(x^2 + 2x) + 5 \quad \text{(Complete square)}$$

[Ans: a=7, b= – 2 c=1**]**

5. Equation of Straight Line

m = **Slope of line** L passing
through points P & Q
= $(b - d)/\ (a - c)$ = tan A



Equation of L**:**
$y - b = m(x - a)$

x**-intercept** of L = e

y **-intercept** of L = f

6. Sketch Graph of Basic Functions

(Also, find their domain and range)

i. $y = \sqrt{x};\ y = x^2;\ x = y^2;\ y = x^3;\ y = |x|$

ii. $y = e^x;\ y = 2^x;\ y = 2^{-x};$

iii. $y = \ln x;\ y = \log_2 x;\ y = \log_{\frac{1}{2}} x$

iv. All trigonometric Functions

7. Sketching Graph [by using (5)]

3-Step Method: i. Identify the basic function
ii. Use Horizontal & Vertical Shifts
iii. Use Reflections about the axes

i. $y = 5 - 4x - 2x^2$ **(Parabola !!!)**

Sol. (a) Complete square on Right Side (See 4)
(b) Basic function $y = kx^2$ (k is a real number)
(c) Make use of Horizontal & Vertical Shifts.

ii. $y = |2x - 3| + 4$

Sol. (a) **Note** $|2x - 3| = 2|x - \tfrac{3}{2}|$

(b) Basic function $y = k|x|$ (k is a real number)

(c) Make use of Horizontal & Vertical Shifts.

iii. $y = |x^2 - x - 6|$

Sol. (a) Sketch Parabola $y = x^2 - x - 6$ by finding the x-intercepts
(b) Reflect the part of graph which below the x-axis.

iv. Sketch the following graphs:

a. $y = (x-2)^3 + 4$ b. $y = \ln(x+2) - 4$
c. $y = e^{-x-3} - 1$ d. $y = |\ln(x+2)| - 4$
e. $y = \ln|x+2| - 4$ f. $y = e^{|-x-3|} - 1$

(Use the 3-step Method explained above)

8. Distance Formula

Two points P and Q lie on the graph of $y = \sqrt{x-1} + 2$ **with 1st coordinates 5 and 10 respectively. Find the distance between P & Q.**

9. Factorization

i. Factorize: $x^3 - 64$; $x^6 - 5x^3 + 6$; $3x^3 - x^2 + 3x - 1$
ii. Check if $x = -2$ is a zero of the polynomial $x^3 - 2x + 4$. If so, factorize it.

10. Use of inequalities

Domain of the function: $y = \sqrt{\dfrac{-x}{1+x}}$ *Ans*: (– 1, 0]

11. Long Division

Divide $5x^4 + 3x^2 - 4x - 5$ by $x^2 - 2x + 4$ and find Divisor; Quotient and Remainder.

12. Zeros (Roots) of Polynomial *p*(*x*)

a. **Check that *x* = *a* is a zero of *p*(*x*)**
Use Remainder Theorem: $p(a) = 0$

b. **Note: Graph of *p*(*x*)**
at a ***Single zero***: crosses x-axis
at a ***Double zero***: does not cross x-axis

c. **Synthetic Division**
Find Quotient and Remainder when
$p(x) = 2x^3 - 3x + 1$ is divided by $x - 3$

d. **Rational Zero Theorem**
Find all possible rational zeros of
$p(x) = 2x^4 - 3x^3 + 4$

13. Inverse of Function

Find the inverse of (i) $f(x) = 2x + 3$, (ii) $f(x) = \sqrt{2x+3}$. Also, find the domain and range of f^{-1}.

14. System of equations

Solve $\begin{aligned} x^2 + y^2 &= 7 \\ 2x + y &= -3 \end{aligned}$

15. Finding vertical & horizontal asymptotes and holes

For $y = \dfrac{x^2 - 2x + 1}{(x-1)(x-2)}$, find

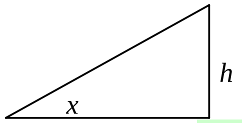
a. its vertical & horizontal asymptotes?
b. the point where the graph has a hole

16. Trigonometric Formulas

i. $\sin(a \pm b)$; [Same for cos & tan]
ii. $\sin(a \pm \pi/2)$; $\sin(a \pm \pi)$;
[Same for cos & tan]
iii. $\sin(2a)$; [Same for cos & tan]
iv. $\sin(a/2)$; [Same for cos & tan]
v. $\sin^2 a + \cos^2 a = 1$; $\tan^2 a + 1 = \sec^2 a$

17. Use of Right Angled Triangle to find Trigonometric Ratios

A. If $\sin x = h$ in the diagram, find



i. $\sec x$; ii. $\sin(2x)$; iii. $\cos\dfrac{x}{2}$

[Recall Half & Double angle formulas]

B. Show that $\sqrt{x^2 - 9} = 3\tan x$ if $x = 3\sec\theta$.
[Use Right Angled Triangle]

18. Geometrical Figures

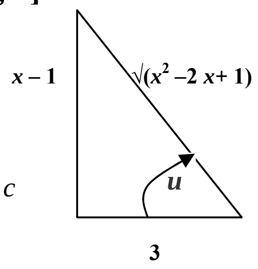
See inside Text Cover Page for all figures and formulas for
Area of Triangle, Trapezoid, Circle; Sector
and
Volume and Surface Area of
Cylinder, Cone and Frustum.

Integration Techniques

Appendix 3(v)

No	Type	Formula	Exercise I	Exercise II
1	**Power Rule**	$\int (ax+b)^m\,dx = \frac{(ax+b)^{m+1}}{a(m+1)} + c;\ m \neq -1$	$\int (2-7x)^{11}dx$	$\int \frac{1}{(-8x+45)^{21}}dx$
2.	**Substitution**	$\int g(f(x))f'(x)dx = \int g(u)du$ where $u = f(x)$	$\int \frac{(x-2x^3)dx}{6(x^4-x^2+7)^{\frac{7}{3}}}$	$\int (x^2-6x)(x^3-9x^2-17)dx$
3	**Direct Formulas exp & log**	i. $\int \frac{1}{x}dx = \ln\lvert x\rvert + c$ ii. $\int e^x dx = e^x + c$ iii. $\int a^x dx = \frac{a^x}{\ln a} + c;\ a > 0$	i. $\int \frac{18}{20x-7}dx$ ii. $\int -71e^{4x+9}dx$ iii. $\int 5\,(2.5)^{-4x+3}\,dx$	i. $\int \frac{3x^2-7x}{2x^3-7x^2+5}dx$ ii. $\int \left(3-8x^3\right)e^{-2x^4+3x}dx$ iii. $\int \frac{5\left(6x-7\right)}{7^{3x^2-7x}}\,dx$
4	**Simple Trig. Function**	i. $\int \sin(ax+b)dx = \frac{-1}{a}\cos(ax+b) + c$ ii. $\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + c$ iii. $\int \sec^2 x dx = \tan x + c$ iv. $\int \csc^2 x dx = -\cot x + c$ v. $\int \tan x \sec x dx = \sec x + c$ vi. $\int \cot x \csc x dx = -\csc x + c$	i. $\int \sin(6x-4)dx$ ii. $\int \cos(8-9x)dx$ iii. $\int \sec^2(3x+7)dx$ iv. $\int \csc^2(2-5x)dx$ v. $\int \tan 4x \sec 4x dx$ vi. $\int \cot \pi x \csc \pi x dx$	i. $\int x^4 \sin(6x^5-4)dx$ ii. $\int \sin^3 5x \cos 5x dx$ iii. $\int \tan^4 8x \sec^2 8x dx$ iv. $\int \cot^3 x \csc^4 x dx$ v. $\int \frac{1}{x^2}\tan\frac{\pi}{x}\sec\frac{\pi}{x}dx$ vi. $\int x^6 \cot x^7 \text{cs}\, x^7 dx$
5	**Use of Inverse Trig. Functions** **Also, see 9**	i. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ ii. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$ iii. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left\lvert\frac{x}{a}\right\rvert + c$	i. $\int \frac{dx}{\sqrt{25-9x^2}}$ ii. $\int \frac{dx}{100+49x^2}$ iii. $\int \frac{dx}{x\sqrt{16x^2-81}}$	i. $\int \frac{dx}{\sqrt{18+8x-x^2}}$ ii. $\int \frac{dx}{170+70x+49x^2}$ iii. $\int \frac{dx}{\left(x-9\right)\sqrt{x^2-18x}}$
5	**Integration by Parts**	Rule: $\int udv = uv - \int vdu$ i. $\int xe^x dx = xe^x - e^x + c;$ $\left[u = x, dv = e^x dx \Rightarrow du = dx, v = \int e^x dx\right]$ ii. $\int x \sin x dx = -x\cos x + \sin x + c$ iii. $\int \ln x dx = x\ln x - x + c;$ $\left[u = \ln x, dv = dx\right]$ iv. $\int \cos xe^x dx = \frac{1}{2}\left(\cos xe^x + \sin xe^x\right) + c$	i. $\int \sin^{-1} x dx$ ii. $\int \log x dx$ iii. $\int x^2 \ln x dx$ iv. $\int x \tan^{-1} dx$ v. $\int x^2 2^x dx$ vi. $\int x^2 \sin x dx$ vii. $\int x^3 \sin x^2 dx$	i. $\int 6xe^{5x-9}dx$ ii. $\int x^7 \cos x^4 dx$ iii. $\int x^4 \ln\left(3x^5-11\right)dx$ iv. $\int \frac{\ln x^2}{x^2}dx$ v. $\int \left(x^2-2x\right)\cos^{-1}(x^3-3x^2)dx$ vi. $\int x \sec^{-1} x dx$ vii. $\int x \tan^{-1} x dx$
6	**More Trig. Formulas**	i. $\int \tan x dx = -\ln\lvert\cos x\rvert + c$ ii. $\int \cot x dx = \ln\lvert\sin x\rvert + c$ iii. $\int \sec x dx = \ln\lvert\sec x + \tan x\rvert + c$ iv. $\int \csc x dx = \ln\lvert\csc x - \cot x\rvert + c$	i. $\int x \tan\left(x^2-7\right)dx$ ii. $\int \frac{\cot\sqrt{x}}{\sqrt{x}}dx$ iii. $\int x \sec x dx$	i. $\int \left(5x-1\right)\sec\left(5x^2-2x\right)dx$ ii. $\int \frac{1}{\left(3x-1\right)^2}\csc\frac{1}{3x-1}dx$

Use of Trig. Identities in Integration

	Trig. Identities	Exercises
7	i. $\cos^2 x = \frac{1+\cos 2x}{2}$ ii. $\sin^2 x = \frac{1-\cos 2x}{2}$ **(See **)** iii. $\sin 2x = 2\sin x\cos x$	i. $\int \sin^2(3x+1)dx$ ii. $\int \sin^4 5x dx$ iii. $\int x\sin^2(7x^2+3)dx$ iv. $\int x\cos^2(3x+1)dx$ v. $\int \cos^3 5x dx$ vi. $\int x\cos(7x^2+3)dx$ vii. $\int \cos^2 3x\sin^2 3x dx$ viii. $\int x^3\sin^2(x^2+3)dx$
8	i. $\sin(a\pm b)=\sin a\cos b\pm\cos a\sin b$; ii. $\cos(a\pm b)=\cos a\cos b\mp\sin a\sin b$; iii. $\sin a\sin b=\frac{1}{2}[\cos(a-b)-\cos(a+b)]$ iv. $\sin a\cos b=\frac{1}{2}[\sin(a+b)+\sin(a-b)]$; v. $\cos a\cos b=\frac{1}{2}[\cos(a+b)+\cos(a-b)]$ **Exercises:** i. $\int\sin 5x\cos 3x dx$ ii. $\int\cos 5x\cos 3x dx$ iii. $\int\sin 5x\sin 3x dx$	
9	**Example: [Completion of Square & Use of Right Triangle]** $I=\int\frac{x-2}{\sqrt{x^2-2x+10}}dx : x^2-2x+10=(x-1)^2+9$; Set $x-1=3\tan u \Rightarrow dx=3\sec^2 u du.$ $I=\int(3\tan u-1)\sec u du=3\sec u-\ln\lvert\sec u+\tan u\rvert+c=\sqrt{x^2-2x+10}-\ln\left\lvert\frac{\sqrt{x^2-2x+10}}{3}+\frac{x-1}{3}\right\rvert+c$  	

Use of Partial Fractions Decomposition of Rational Functions *N*(*x*)/*D*(*x*) in Integration

[***Note:** If deg (***N***) ≥ deg (***D***), first divide ***N*** by ***D***. Then find Partial Fractions Decomposition.]

Here, in (i-iv), we assume that **deg** (Numerator) < **deg** (Denominator)

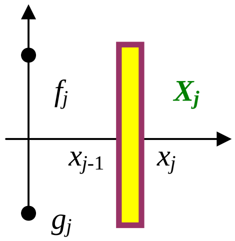
10	Type	Method	Exercises
i	**Non-Repeated Linear Factors**	$\frac{p(x)}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}$ [Write: ***p*(*x*)=*A*(*x-b*)+*B*(*x-a*)**]	$\frac{x+11}{x^2-2x-15}=\frac{A}{x+3}+\frac{B}{x-5}$; Ans: $A=-1,\ B=2$
ii	**Repeated Linear Factors**	$\frac{p(x)}{(x-a)(x-b)^2}=\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{(x-b)^2}$	$\frac{x^2+2x+7}{x(x-1)^2}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$; Ans: $A=7,\ B=-6, C=10$
iii	**Non-Repeated Non-Linear Factors**	$\frac{p(x)}{(x^2+bx+c)(x^2+dx+g)}=\frac{Ax+B}{x^2+bx+c}+\frac{Cx+D}{x^2+dx+g}$	$\frac{3x+16}{(x-2)(x^2+7)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+7}$; Ans: $A=2,\ B=-2, C=-1$
iv	**Repeated Non-Linear Factors**	$\frac{p(x)}{(x-a)(x^2+bx+c)^2}=\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}+\frac{Dx+E}{(x^2+dx+g)^2}$	$\frac{4x^3+5x^2+7x-1}{(x^2+x+1)^2}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{(x^2+x+1)^2}$; Ans: $A=4,\ B=1, C=2, D=-2$
* v	deg (***N***) ≥ deg (***D***)	$\frac{N(x)}{D(x)}=\frac{N(x)}{(x-a)(x-b)}=Q(x)+\frac{R(x)}{(x-a)(x-b)}$; where $\frac{R(x)}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}$	$\frac{x^3-4x^2-19x-35}{x^2-7x}=Q(x)+\frac{R(x)}{x^2-7x}$; where $\frac{R(x)}{x^2-7x}=\frac{A}{x}+\frac{B}{x-7}$; Ans: $Q(x)=x+3$; $R(x)=2x-35$; $A=4,\ B=1, C=2, D=-2$
**	**Trig. Identities:** $\cos^2\theta+\sin^2\theta=1$; $1+\tan^2\theta=\sec^2\theta$; $1+\cot^2\theta=\csc^2\theta$		

I. i. Area of Rectangle

Height: $f_j - g_j$

Width: $\Delta x_j = x_j - x_{j-1}$

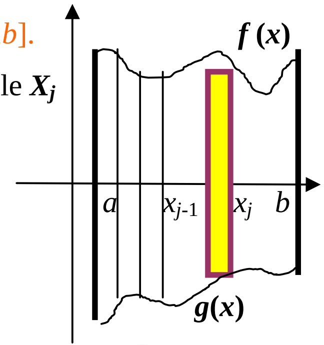
Area: $(f_j - g_j)\Delta x_j$



ii. Area of Region *R* bounded above by *f* (*x*) & below by *g*(*x*), *x* in [*a*,*b*].

a. Area of *j*th Rectangle X_j

$=[f(x_j)-g(x_j)\Delta x_j$



b. **Area** of Region ***R***

$$= \lim_{\max(\Delta x_j \to 0)} \left[\sum_j [f(x_j)-g(x_j)]\Delta x_j \right]$$

$$= \int_a^b [f(x)-g(x)]dx$$

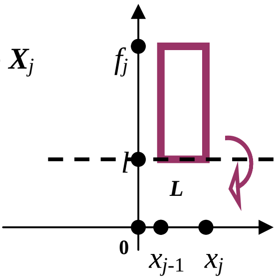
II i. Disc: Given in the *xy*-plane

X_j

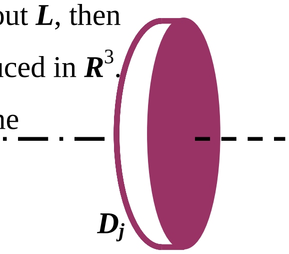
a. a **Vertical** Rectangle X_j

b. a **Horizontal** line ***L***

c. One side of X_j is connected to ***L***.



When X_j is Revolved about ***L***, then a Solid object D_j is produced in R^3. The object D_j is Called the **Disc** with centre on *L*.



ii . Volume of Disc D_j .

Consider the Rectangle X_j in the above figure.

Note that

a. Distance (Upper Edge of X_j , *x*-axis) = f_j

b. Distance (***L***, *x*-axis) = l

c. Width of X_j = $\Delta x_j = x_j - x_{j-1}$

Then

a. Height of X_j = $f_j - l$ (Radius of D_j)

b.Width of X_j = $\Delta x_j = x_j - x_{j-1}$ (Thickness of D_j)

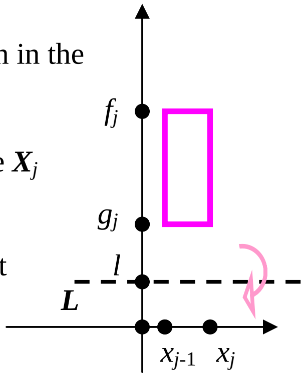
c. Volume of D_j = $\pi(f_j - l)^2\Delta x_j$

III i. Washer: Given in the *xy*- plane

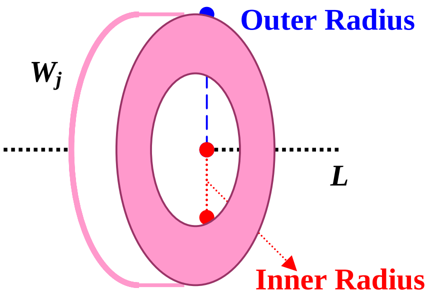
a. a **Vertical** Rectangle X_j

b. a **Horizontal** line ***L***

c. Any side of X_j is not connected to L.



When X_j is Revolved about ***L***, then a Solid object W_j produces in R^3. The object W_j is Called the **Disc** with centre on *L*.



ii. Volume of Washer W_j .

Consider the Rectangle X_j in the figure (**3 i**).

Note that

i. Distance (Upper Edge of X_j , *x*-axis) = f_j

ii. Distance (Lower Edge of X_j , *x*-axis) = g_j

ii. Distance (***L***, *x*-axis) = l

iii. Width of X_j = $\Delta x_j = x_j - x_{j-1}$

Then

i. Outer Radius of W_j = $f_j - l$

ii. Inner Radius of W_j = $g_j - l$

ii.Width of X_j $= \Delta x_j = x_j - x_{j-1}$

= Thickness of W_j

iii. Volume of W_j

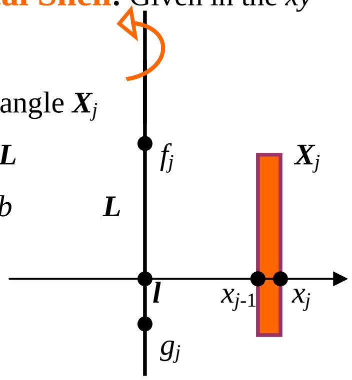
$$= \pi[(f_j - l)^2\text{-}(g_j - l)^2]\Delta x_j$$

IV i. Cylindrical Shell: Given in the *xy*-plane

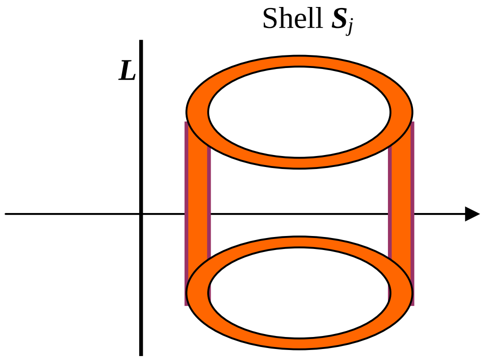
a. a **Vertical** Rectangle X_j

b. a **Vertical** line L

c. lines $x = a$, $x = b$



When X_j is Revolved about L, then a Solid object S_j is produced in R^3. The object S_j is called the **Cylindrical Shell.**



ii. Volume of Cylindrical Shell S_j .

Consider the Rectangle X_j in the figure (**XV**).

Note that

a. Distance (Upper Edge of X_j , x-axis) = f_j

b. Distance (Lower Edge of X_j , x-axis) = g_j

c. Width of X_j = $\Delta x_j = x_j - x_{j-1}$

Then

a. Outer Radius of S_j = $x_j - l$

b. Inner Radius of S_j = $x_{j-1} - l$

c.Ave. Radius of S_j $=[\dfrac{x_j + x_{j-1}}{2} - l]$

d.Width of X_j $= \Delta x_j = x_j - x_{j-1}$

= Thickness of S_j

e. Volume of S_j

$$=(f_j - g_j)\ \pi\, [(x_j - l)^2 - (x_{j-1} - l)^2]$$

$$= \pi (f_j - g_j) [(x_j - x_{j-1})(x_j + x_{j-1} - 2l)]$$

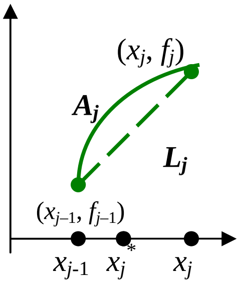
$$= \pi (f_j - g_j) [x_j + x_{j-1} - 2l]\ \Delta x_j$$

$$= 2\pi (f_j - g_j) [\frac{x_j + x_{j-1}}{2} - l]\ \Delta x_j$$

$$= 2\pi (f_j - g_j) (x_j^* - l) \Delta x_j$$

[$x_j^* - l$ is called **Average Radius of Shell**]

V i. Arc Length:



Given:

a. A small Arc A_j with end points (x_{j-1}, f_{j-1}) and (x_j, f_j)

b. The line segment L_j joining the points $(x_{j-1}, f(x_{j-1}))$ and $(x_j, f(x_j))$.

Length of Arc A_j ≈ Length of Segment L_j

if $\Delta x_j = x_j - x_{j-1} \approx 0.$

ii. Length of Segment L_j

$$= \sqrt{[x_j - x_{j-1}]^2 + [f(x_j) - f(x_{j-1})]^2}$$

$$= \sqrt{[x_j - x_{j-1}]^2 + [(x_j - x_{j-1}) f(x_j^*)]^2}$$

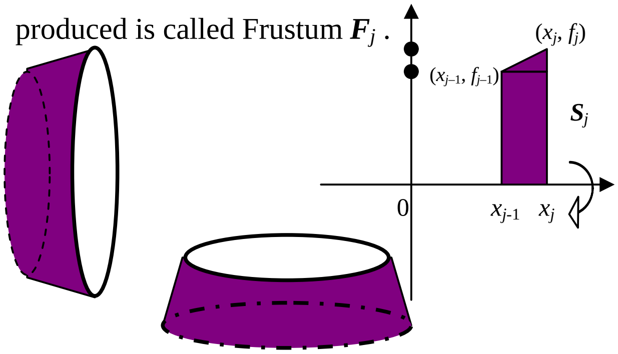
(***By Mean Value Theorem***)

$$= \sqrt{1 + (f'(x_j^*))^2}\ \Delta x_j \approx \textbf{Length of Arc } A_j$$

IV i. Frustum F_j : Given in the *xy*-plane: “

A small Slant Segment L_j joining the points $(x_{j-1}, f(x_{j-1}))$ and (x_j, f_j) ”.

When L_j is Revolved about x-axis, the surface produced is called Frustum F_j .



Formula: *Surface Area of Frustum.=* $\pi\, l\, (r + R)$

iii. Surface Area of Frustum F_j :

$$= \pi \sqrt{(x_j - x_{j-1})^2 + [f(x_j) - f(x_{j-1})]^2}\{f(x_j) + f(x_{j-1})\}$$

$$= \pi \sqrt{1 + \left[\frac{f(x_j) - f(x_{j-1})}{x_j - x_{j-1}}\right]^2}\{f(x_j) + f(x_{j-1})\}(x_j - x_{j-1})$$

$$= 2\pi \sqrt{1 + \underbrace{(f'(x_j^*))^2}_{\text{By Interm Value Theorem}}}\ \Delta x_j \underbrace{\left(\frac{(f(x_j) + f(x_{j-1})}{2}\right)}_{\text{Average Height of Trapezium}}$$

$$= 2\pi f(x_j^{**}) \sqrt{1 + (f'(x_j^*))^2}\ \Delta x_j$$

King Fahd University of Petroleum and Minerals *Appendix* **3(vii)**
Department of Mathematical Sciences

SYLLABUS & Policies

Semester II, 2004-2005 (042)

Course #: Math 102
Title: Calculus II
Textbook: Calculus (Early Transcendentals) by H. Anton, I. Bivens and S. Davis, 7th edition, 2002.

Course Description: Definite and indefinite integrals. Fundamental Theorem of Calculus. Techniques of Integration. Hyperbolic functions. Applications of integration. Improper integrals. Sequences and series: convergence tests. Alternating series. Absolute and conditional convergence. Power series. Taylor and Maclaurine series.

Week	Date	Sec. #	Topics
1	Feb. 12-16	6.1 6.2	An Overview of the Area Problem The Indefinite integral: Integral Curves
2	Feb. 19-23	6.3 6.4	Integration by Substitution Sigma Notation: Area as a Limit
3	Feb. 26-March 2	6.5 6.6	The Definite Integral The Fundamental Theorem of Calculus
4	March 5-9	6.8 6.9 8.8	Evaluating Definite Integrals by Substitution Logarithmic Functions from the integral Point of View Improper integrals of the form $\int_1^{\infty} f(x)dx$
Note: Chapters 7 & 8 will be covered after the presentation of Chapter 10			
5	March 12-16	10.1 10.2 10.3	Maclaurine and Taylor Polynomial Approx. (till p. 644) Sequences Monotone Sequences
6	March 19-23	10.3 10.4 10.5	Monotone Sequences (Continued) Infinite Series Convergence Tests
7	March 26-30	10.5 10.6 10.7	Convergence Tests (Continued) The Comparison, Ratio and Root Tests Alternating Series Test; Conditional Convergence
Midterm Exam: April 3 (Sunday) 6:30-8:30 pm [Building 10]			
8	April 2-6	10.7 10.8 10.10	Alternating Series; Conditional Convergence (Continued) Maclaurin and Taylor Series; Power Series Differentiating and Integrating Power Series
MIDTERM BREAK: Thursday, April 7—Friday, April 15, 2005			
9	April 16-20	7.1 7.2	Area Between Two Curves Volumes by Slicing: Disks and Washers
10	April 23-27	7.3 7.4	Volumes by Cylindrical Shells Length of a Plane Curve
11	April 30- May 4	7.5 7.8	Area of a Surface of Revolution Hyperbolic Functions and Hanging Cables(pp. 509-513 only)
12	May 7-11	8.2 8.3	Integration by Parts (Please go over sec. 8.1 before starting 8.2) Trigonometric Integrals
13	May 14-18	8.4 8.5	Trigonometric Substitutions Integrating Rational Functions by Partial Fractions
14	May 21-25	8.6 8.8	Special Substitutions (pp. 558-560 only) Improper Integrals
15	May 28-June 1	-	Review and/or catching up

The students are advised to go through the following Text Exercises and consult the Instructor incase of any difficulty in solving any exercise.

Sec. #	**Suggested Practice Problems**
6.1	2,3,10,14, 8,13
6.2	1(a),12,23,30,32,41(c),46,55(a), 2(a),15,27,33,48,55(b)
6.3	2(a),6(a),8,19,28,38,48,50,53(c),57,62, 6(d),16,26,46,54(b),55,67
6.4	1(e),2(c),5,10(a),19,25,28(a),41,49, 6,10(b),14,24,43,60
6.5	1,6,9(b),11(a),13(d),19,22(a),23, 8,14(d),20,22(b),26
6.6	5,18,29(a),31(a),39,50,54,59(a),61, 8,24,30(b),40,55,60(b)
6.8	4,17,20,23,32,37,45,55(c),70(a), 18,21,26,30,42,50,55(a),69
6.9	1(c),3(a,c),16(b),17,21(a),25,34,43, 3(b,d),22,27,30
10.1	3,7,14,22,23,25,34, 11,18,21,26,35
10.2	2,6,10,11,20,21,26,30,31,37,40, 8,12,16,22,36,43
10.3	5,10,15,23,27, 11,17,22,28,30
10.4	1,3,10,13,17,23(a),24(b),25(c),27,30, 2(a),9,14,20,25(a),33
10.5	1,4(a,d),7(b),12,16,25,29(a,c), 2(b),5(d),21,29(b)
10.6	3(a),4(a),9,11,19,32,38,43, 2,8,28,44,51
10.7	1,6,9,14,22,26,33,37,46, 12,20,30,32,36
10.8	2,5,12,15,21,24,29,35,44,50, 10,16,23,38,47
10.10	1(c,d),6(d),7(a),9(a),14(a),25,27(a),33(a),34(a), 2(a),5(d),11,15(b),26,33(b),34(b)
7.1	3,6,9,14,15,32,33,44, 4,8,18,31
7.2	3,11,15,24,25,28,31,39(a), 8,20,30,33,39(b)
7.3	4,12,16,21,27, 10,20,26
7.4	4,6,9,12, 8,13
7.5	2,8,24, 18,23
7.8	3(b,d),4(a,c),12,15,33,37,51,64(b),67, 3(a,c),17,36,56(f),63(c)
8.2	2,7,14,18,23,27,37,41(a),46,54(a), 12,20,28,38,41(b),47,57(b)
8.3	8,11,21,30,41,51,55, 15,32,44,50,64
8.4	3,8,20,24,29,38,41, 4,25,31,45
8.5	2,8,9,14,16,25,30,33, 7,13,32,36
8.6	56,59,66,69,72, 58,71
8.8	1(b,c),2(a,b),4,12,17,26,29,31,41,44,62, 2(c),5,30,33,49

Evaluation: Grades in this course are based upon the point total of Midterm Exam, quizzes, Homework assignments and a *comprehensive* final exam (worth 200 points). The following grading scale can be used to estimate grades for individual quizzes and exams:

90%	–	100%	4.0	65%	–	69.9%	2.0
85%	–	89.9%	3.5	60%	–	64.9%	1.5
78%	–	84.9%	3.0	55%	–	59.9%	1.0
70%	–	77.9%	2.5				

There will be no "make–ups" for exams or quizzes *unless a valid excuse is presented* in advance.*, A missed exam or quiz will receive the score 0.* Students must look at this syllabus carefully and ***plan well ahead***.

Homework: Weekly Homework problems will be displayed on instructors website every Tuesday-Wednesday. Students must do the homework according to the instructions. Homework should be submitted every Monday in the class.

You are encouraged to visit my office hours or make an appointment to discuss any difficulties related to the course, including the homework problems.

Remember:

"**The best way to learn Mathematics is to do Mathematics."**

Attendance: KFUPM policy with regard to attendance (**lectures and recitations**) will be enforced. Students are expected to attend all class meetings and are responsible for all of the material covered.

Changes: Any changes in the syllabus or in the scheduling of exams, quizzes, etc. will be announced during class meetings. Students who miss a class meeting should consult a classmate and also copy his notes for that meeting.

Guideline to Solve Applied Maxima / Minma Problems

1. **Draw a diagram suitable for the Problem.**
2. **Identify the known and unknown quantities.**
3. **Label the unknown quantities.**
4. **Identify the variable (in Step 3) that is to be Maximized or Minimized.**
5. **Find the Relation among the Variables / Known Quantities of the problem**
6. **Find the Additional /Side conditions, if any.**
7. **Convert the Main Equation to Equation of 1 variable using Step 6, if required**
8. **Find the interval suitable for the variable to be Maximized or Minimized.**
9. **Find the Critical Numbers of the function in Step 7.**
10. **Check the C.N. (in Step 9) and also the end points of the Interval (in Step 8) for Relative Extremum.**

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Ex 5.3

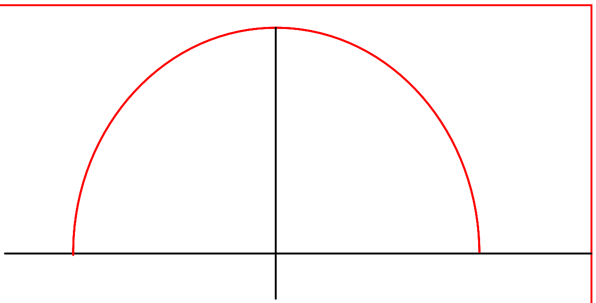
Q 2. How should 2 nonnegative numbers be chosen so that their sum is 1 and sum of their squares is (a) as small as possible (b) as large as possible.

Solution:

1. Identify the Unknowns:	**2 nonnegative numbers;** **Sum of Squares of x & y**	
2. Label the Unknowns:	**x & y =2 nonnegative integers;** **z = Sum of Squares of x & y**	
3. Identify the Variable to be Maximized/Minimized:	z	**Interval for x & y** **[0, 1]**
4. Relation among the Variables		**• Find C.N of z** **• Test the C.N. and 0,1 for Max/Mini Values**
5. Side Conditions:	$x + y = 1$	
6. Convert Main Eq. To Eq. of One Variable:	$z = x^2 + (1- x)^2$	**z is Minimum: $x = y = ½$** **z is Maximum: $x =1$, $y = 0$**

Q 8. A rectangle has its 2 lower corners on the x-axis and 2 upper corners on the curve $y = 16 - x^2$. For all such rectangles, what are the dimensions of the one with largest area?

1. **Draw the Diagram:**



2. **Identify the Unknowns:**

i.Coordinates of upper corners of rectangle
ii. Length & Width of Rectangle
iii. Area of Rectangle

3. **Label the Unknowns:**

$B = (-x,0)$ **$C = (x,0)$**
$D = (x, 16 - x^2)$ **$E = (-x, 16 - x^2)$**
$L = 2x$ **$W = 16 - x^2$**
A=Area of Rectangle

4. **Identify the Variable to be Maximized/Minimized:** **A**

5. **Relation among the Variables** **A = L W**

6. **Side Conditions:** $0 \leq x \leq 4$

7. **Convert Main Eq. To Eq. of One Variable:**

A = L W
$= 2x\,(16 - x^2)$
$= 32x - 2x^3$
$A' = 32 - 6x^2$
C.N:
$x = 4/\sqrt{3}$
$A'' = -12x$

Interval for x & y
[0, 4]

- **Find C.N of A**
- **Test the C.N. and 0,4 for Max/Mini values**

A is Max. at $x =$ $4/\sqrt{3}$

$\Rightarrow L=2x= 8/\sqrt{3}$

$W=16-x^2=32/3$

Specimen Form for Solving Applied Minima / Maxima Problems [MATH 101]

Analyze the Word Problem by Filling in the following Table

No.	Item	Analysis
1	**Draw the Diagram** (if required)	
2	**Identify and Label** the unknowns	
3	Identify the Variable to be Optimized (Maximized or Minimized)	
4	Relation among the Variables (May need Geometrical Formula)	
5	**Side Conditions** a) Relation between the variables [*other than the variable to be optimized*] b) Any Restriction on the Range of Variables.	
6	**Convert Main Equation** To Equation of One Variable (Use (5) in (4))	
7	**Find Critical Number(s)** of the Variable to be Optimized	
8.	**Use 2nd Derivative to Test** Stationary Points for Local Extrema (if Required)	
9	**Form a Table to Test** a) all C.N. b) the numbers restricting the Independent Variables in (6) **for** **Absolute Maxima Minima**	

Classroom Problems

(Use the above Table to solve the following problems)

Q 19. An open box is to be made from a 3 ft by 8 ft rectangular piece of sheet metal by cutting out squares of equal size from the 4 corners and bending up the sides. Find the maximum volume that the box can have.

Q 26. Show that the right circular cylinder of greatest volume that can be inscribed in a right circular cone has volume that is 4/9 the volume of the cone.

Q 38. A trapezoid is inscribed in a semicircle of radius 2 so that one side is along the diameter. Find the maximum possible area of the trapezoid.

Q. Find all points on the curve $x^2 - y^2 = 1$ closest to (0,2).

MATH 102 **[Homework 11:** Due on **Wed, May 11]**

Q1. Find the volume of the solid by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer:

i. $x = y - y^2, x = 0$; about $y = 0$.
ii. $y = x^{2/3},\ x = 1,\ y = 0$; about $y -$ axis.

Q2. Find the volume of a frustum of a pyramid with square base of side 4, square top of side 1 and height 6. (Sketch the diagram. Use the method of cross section)

Q3. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

i. $y = 0,\ y = \sin x, 0 \le x \le \pi$; about $y = 1$.
ii. $2x + 3y = 6,\ (y-1)^2 = 4 - x$; about $y = -5$.

Q4 . (*Review*) Find the limit of the sequence**:**

i. $\left\{\frac{\ln n}{\ln 2n}\right\}_{n=2}^{\infty}$ ii. $\left\{\frac{1.2....(2n-1)}{(2n)^n}\right\}_{n=2}^{\infty}$

Q5. (*Review*) Find the sum of the series: $\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n}$