Section 15.7

(Part 1)
Triple integrals in cylindrical coordinates

Learning outcomes

After completing this part, you will inshaAllah be able to

1. evaluate triple integrals in cylindrical coordinates
2. convert triple integrals from rectangular coordinates to cylindrical coordinates.

Recall

- Cylindrical coordinates: (r,θ,z)
- Conversion formulas for cylindrical coordinates

$$x = r\cos\theta,\quad y = r\sin\theta,\quad z = z$$

Aim: Learn to integrate $f(r,\theta,z)$

- **Recall** area element in polar coordinates: $dA = r\,dr\,d\theta$
- Volume element in cylindrical coordinates $dV = r\,dz\,dr\,d\theta$

Evaluating triple integrals in cylindrical coordinates

Given a function $f(r,\theta,z)$ over a soild G such that

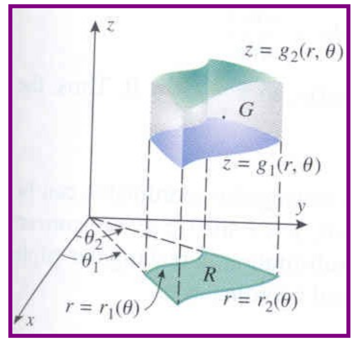
- G is bounded above by $z = g_2(r,\theta)$ and below by $z = g_1(r,\theta)$
- and the projection R of G on XY-plane is a simple polar region.

Then the triple integral is evaluated as follows:

$$\iiint_G f(r,\theta,z)dV = \iint_R \left[\int_{g_1(r,\theta)}^{g_2(r,\theta)} f(r,\theta,z)dz \right] dA$$

or

$$\iiint_G f(r,\theta,z)dV = \int_{\theta_1}^{\theta_2} \int_{r_1(\theta)}^{r_2(\theta)} \int_{g_1(r,\theta)}^{g_2(r,\theta)} f(r,\theta,z)dzdrd\theta$$



Example 15.7.1

Evaluate $\iiint_G r\sin\theta dV$ where G is the region that lies below the plane $z = r\cos\theta + 2$, above the XY-plane and between the cylinders $r = 1$ and $r = 2$.

Solution Done in class

Example 15.7.2 See class notes for the example & solution

Converting triple integrals from rectangular to cylindrical coordinates

Some triple integrals are easier to evaluate in cylindrical coordinates.

Especially,

- If the expressions of the form $x^2 + y^2 = a^2$ appear in the limits or integrand

How to convert?

- Use $\begin{matrix} x = r\cos\theta \\ y = r\sin\theta \end{matrix} \Rightarrow \quad x^2 + y^2 = r^2$
- and convert limits

Example 15.7.3 Convert $I = \int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\,dz\,dx\,dy$ into an integral in cylindrical coordinates.

Solution: Done in class

Important steps for conversion

- identify the regions G, R
- then find limits of integration in cylindrical coordinates

End of 15.7 (PART 1)

Section 15.7

(Part 2)
Triple integrals in spherical coordinates

Learning outcomes

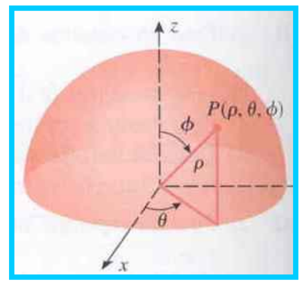
After completing this part, you will inshaAllah be able to

1. evaluate triple integrals in spherical coordinates

See class explanation

Recall

- Cylindrical coordinates: (ρ, θ, ϕ)
- $\rho = \text{constant}$: sphere
- $\phi = \text{constant}$: cone



Aim: Learn to integrate $f(\rho, \theta, \phi)$

Volume element in cylindrical coordinates $dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

Evaluating triple integrals in spherical coordinates

Evaluated as triple integral

$$\iiint_G f(\rho,\theta,\phi)\,dV = \iiint_{\text{limits}} f(\rho,\theta,\phi)\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

We learn the techniques of putting limits through examples

Example 15.7.4 Evaluate $\iiint_G 16\rho\cos\phi\, dV$ where G is the upper half of sphere $x^2+y^2+z^2=1$.

Solution Done in class

Example 15.7.5 Evaluate $V=\iiint_G dV$ where G is the bounded by $\phi=\dfrac{\pi}{6}$, $\rho=3$.

Solution Done in class

End of 15.7 (PART 2)

End of the course

You have the license to steer all models of Math201