

## Section 15.1 *Double integrals*

15.1<sub>1</sub>

### Learning outcomes

After completing this section, you will inshaAllah be able to

1. understand what is meant by **double integral of  $f(x, y)$**
2. **evaluate double integrals of  $f(x, y)$  over rectangular regions**
3. **apply double integration to find volume** under a non-negative function  $f(x, y)$ .

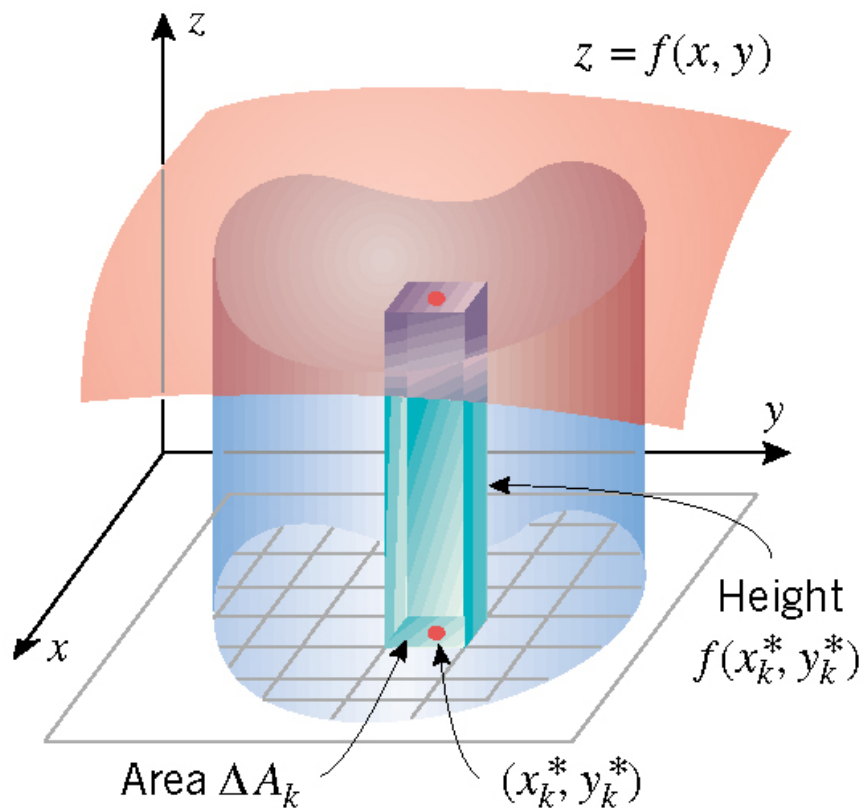
**Motivation:****Volume under a surface  $z = f(x, y) \geq 0$** 

❖ In Math101, we did

area problem -----> definite integration of  $f(x)$

❖ Here we see that

volume problem -----> double integration over a region  $R$



- Consider a non-negative function  $z = f(x, y)$  over a region  $R$

- Approximate the volume under  $z = f(x, y)$ , by adding the volumes of cubes {see figure for understanding}

- Volume of the  $k^{\text{th}}$  cube =  $f(x_k^*, y_k^*)\Delta A_k$

- Approximate volume =  $\sum_{k=1}^n f(x_k^*, y_k^*)\Delta A_k$

- Exact volume =  $\lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k^*, y_k^*)\Delta A_k$

See class  
explanation

## Double integrals & basic properties

The double integral of a function  $f(x, y)$  over a region  $R$  is defined as

$$\iint_R f(x, y) dA = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k^*, y_k^*) \Delta A_k$$

If  $f(x, y) \geq 0$  on  $R$  then the volume under  $f(x, y)$  over the region  $R$  is given by

$$\iint_R f(x, y) dA$$

The integral  $\iint_R 1 \cdot dA = \iint_R dA$  gives area of the region  $R$

### Basic properties of double integrals

- $\iint_R cf(x, y) dA = c \iint_R f(x, y) dA$
- $\iint_R [f(x, y) \pm g(x, y)] dA = \iint_R f(x, y) dA \pm \iint_R g(x, y) dA$
- $\iint_R f(x, y) dA = \iint_{R_1} f(x, y) dA + \iint_{R_2} f(x, y) dA$

### Main Question:

Efficient methods of evaluating double integrals

Computing them by definition is too difficult and impractical

## Evaluating double integrals over rectangular regions

more general regions in next section

If the region  $R$  is defined by  $a \leq x \leq b$ ,  $c \leq y \leq d$

then

$$\iint_R f(x, y) dA = \int_c^d \int_a^b f(x, y) dx dy = \int_a^b \int_c^d f(x, y) dy dx$$

i.e.  $R$  is rectangular  
region  $[a, b] \times [c, d]$

**Note:**

- inner limits for inner differential
- outer limits for outer differential

These integrals are evaluated as iterated integrals

$$\int_c^d \int_a^b f(x, y) dx dy = \int_c^d \left[ \int_a^b f(x, y) dx \right] dy$$

$$\int_a^b \int_c^d f(x, y) dy dx = \int_a^b \left[ \int_c^d f(x, y) dy \right] dx$$

See examples 15.1.1, 15.1.2, 15.1.3 done in class

Do whole exercise

End of 15.1