# Section 14.9 *Lagrange multipliers*



## Learning outcomes

After completing this section, you will inshaAllah be able to

1. understand what is meant by constrained optimization problem
2. use method of Lagrange multipliers to solve optimization problems for $f(x, y)$ subject to one constraint $g(x, y) = c$

# Constrained extrema for *f(x,y)* (with one constraint)

Also called finding constrained extrema

**Question:**

To optimize a function $f(x,y)$

subject to a given constraint $g(x,y)=c$

e.g. the constraint can be boundary.

**Answer: Lagrange theorem**

If $f(x,y)$ has an extrema at $(x_0,y_0)$ subject to $g(x,y)=c$ then $\nabla f(x_0,y_0)=\lambda\nabla g(x_0,y_0)$

$\lambda$ is called Lagrange multiplier

Note

- The solutions of this equation gives all possible candidate for extreme points

**Next**

We learn how to use Lagrange theorem to solve constrained optimization problems

## Solving constrained extrema problems

### Method of Lagrange Multipliers

- Given $f(x,y)$ and $g(x,y)=c$
- To find extrema of $f(x,y)$ subject to $g(x,y)=c$.

**STEP 1: Solve the equations**

$$\begin{aligned} \nabla f(x,y) &= \lambda \nabla g(x,y) \\ g(x,y) &= c \end{aligned}$$

OR equivalent equations

$$\begin{aligned} f_x &= \lambda g_x \\ f_y &= \lambda g_y \\ g(x,y) &= c \end{aligned}$$

- Three equations and three unknowns $x,\ y,\ \lambda$
- Can be solved to get $x,\ y$ and $\lambda$

**STEP 2: Examine the points (x,y) obtained from Step 1 for extrema**

**Example 14.9.1** Find the absolute maximum and minimum of $f(x,y)=5x-3y$ subject to constraint $x^2+y^2=136$.

**Example 14.9.2** Find the point on the line $2x-4y=3$ that is closest to the origin.

**Both Solutions** Done in class.

*HW Qs: 5, 8, 14, 17.*

*End of 14.9*