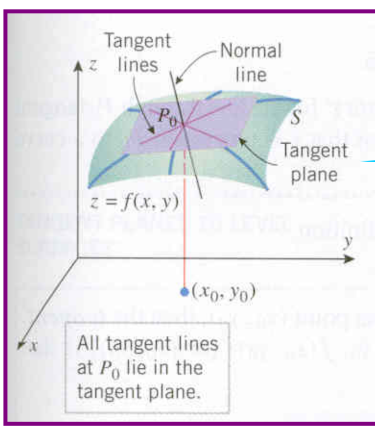
Section 14.7 *Tangent Planes and Normal Vectors*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. find equation of tangent plane and normal line to a surface
2. determine tangent line to the curve of intersection of two surfaces

What are tangent planes & normal lines?



See class explanation

Equation of tangent plane & normal line to a surface

- Given a surface $z = f(x,y)$, we can write it as $F(x,y,z) = c$.
  - i.e. we can think of the surface $z = f(x,y)$ as a level surface $F(x,y,z) = c$ of a function of three variables.
  - Hence, $\nabla F(x_0, y_0, z_0)$ will be normal to the surface $F(x,y,z) = c$ or $z = f(x,y)$ at (x_0, y_0, z_0).

See section 14.6

Given the surface $F(x,y,z) = c$.

- The vector $\nabla F(x_0, y_0, z_0)$ is normal to surface at (x_0, y_0, z_0).
- The equation of the tangent plane at (x_0, y_0, z_0) is

$$F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0) = 0$$

- The equation of the normal line at (x_0, y_0, z_0) is

$$x = x_0 + F_x(x_0,y_0,z_0)t, \quad y = y_0 + F_y(x_0,y_0,z_0)t, \quad z = z_0 + F_z(x_0,y_0,z_0)t$$

What do we need for equation of plane and equation of line

Example 14.7.1 Find the equation of tangent plane and normal line to the surface $x = y^2 + z^2 - 2$ at $(-1,1,0)$.

Note: to write Eq. in form $F(x,y,z) = c$

Solution Done in class

Example 14.7.2 Show that the equation of tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at (x_0, y_0, z_0) can be written in the form $\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} + \frac{z_0 z}{c^2} = 1$.

Solution Done in class

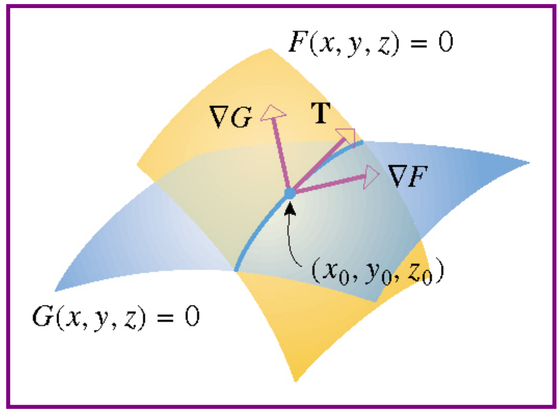
Example 14.7.3 Find the point on the surface $z = 3x^2 - y^2$ at which the tangent plane is parallel to the plane $6x + 4y - z = 5$.

Solution Done in class

Using gradient to find tangent lines to curves of intersection of two surfaces

Let $F(x, y, z) = 0$, $G(x, y, z) = 0$ be two intersecting surfaces & (x_0, y_0, z_0) be a point on the curve of intersection.

- Then both $\nabla F(x_0, y_0, z_0)$ and $\nabla G(x_0, y_0, z_0)$ are normal to the curve of intersection.
- This implies $\vec{\mathbf{v}} = \nabla F \times \nabla G$ is parallel to the tangent line to the curve of intersection.
- Hence, we can write equation of the tangent line to the curve of intersection.



Example 14.7.4 Find the equation of tangent line to the curve of intersection of $z = x^2 + y^2$ and $x^2 + 4y^2 + z^2 = 9$ at $(1 -, 1, 2)$.

Solution Done in class

Do Qs. 1-30

End of Section 14.7