Section 14.4 *Differentiability, local linearity and differentials*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. know how to check differentiability of $z=f(x,y)$
2. learn how to approximate z=f(x,y) using local linear approximation
3. find differential of a function of two or more variables

Checking differentiability of *z=f(x,y)*

A function $f(x,y)$ is differentiable at the point (x_0,y_0)

if $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are continuous at (x_0,y_0).

Local linear approximation of *z=f(x,y)*

If $f(x,y)$ is differentiable (x_0,y_0) then $f(x,y)$

is approximately given by the linear function

$$L(x,y)=f(x_0,y_0)+f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0)$$

see class explanation

Called local linear approximation of f at (x_0,y_0)

- i.e. near (x_0,y_0) we have $f(x,y)\approx L(x,y)$

Gives a good approximation only for (x,y) near (x_0,y_0)

Examples 14.4.1

(a) Show that $f(x,y)=xe^{xy}$ is differentiable at $(1,0)$.

(b) Find local linear approximation of $f(x,y)=xe^{xy}$ at $(1,0)$.

(c) Use this local linear approximation to approximate $f(1.1,-0.1)$

Solution Done in class

Both of above ideas can similarly be extended to functions of more variables

Total differential

- Total differential of $z = f(x, y)$

$$dz = f_x dx + f_y dy = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

Also written as df

- Total differential of $w = f(x, y, z)$

$$dw = f_x dx + f_y dy + f_z dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

Also written as df

Examples 14.4.2 If $z = 3x^2 - xy$, find dz.

Solution Done in class.

End of Section 14.4

Do Qs. 5-8, 25-35.