

Section 14.1 *Functions of two or more variables*

14.1₁

Learning outcomes

After completing this section, you will inshaAllah be able to

1. understand the **definition of functions** $f(x, y)$ **of two variables** and
 - a. learn how to find **domain** of $f(x, y)$
 - b. learn how to **sketch** $f(x, y)$
 - c. know what are **level curves** of $f(x, y)$
2. understand the **definition of functions** $f(x, y, z)$ **of three or more variables**
and
 - a. learn how to find **domain** of $f(x, y, z)$
 - b. know what are **level surfaces** of $f(x, y, z)$

Functions of two variables

Definition

Let $D \subseteq \mathbb{R}^2$. A function of two variables is a rule that assigns a unique real number $f(x, y)$ to each point $(x, y) \in D$.

The set D is called **domain** of $f(x, y)$.

Example 14.1.1 For $f(x, y) = x \ln(y^2 - x)$

(a) Find $f(3, 2)$ (b) Find domain of $f(x, y)$ (c) Sketch domain of $f(x, y)$

Solution Done in class

Exercise

Find domain of $f(x, y) = \frac{\sqrt{x+y+1}}{x-1}$

Answer: $\{D = (x, y) \text{ such that } x + y + 1 \geq 0 \text{ \& } x \neq 1\}$

Graph of $f(x, y)$:

The graph of $f(x, y)$ is the set of all points (x, y, z) satisfying $z = f(x, y)$

Similar to graph of $f(x)$

A surface lying above or below its domain

Example 14.1.2 Sketch the graph of $f(x, y) = 6 - 3x - 2y$.

Solution Done in class

Example 14.1.3 Find the domain of $f(x, y) = \sqrt{9 - x^2 - y^2}$ and sketch the graph of $f(x, y)$.

Solution Done in class

Level curves of $f(x, y)$

- Consider the graph of $f(x, y) = x^2 + y^2$ or $z = x^2 + y^2$
 - It is a circular paraboloid See graph in class
 - At each height level 'k' or 'z=k' we get a circle $x^2 + y^2 = k$ or $f(x, y) = k$

• So, in general, the graph of $f(x, y)$ can be thought as piecing together curves sitting at different height levels.

• The projection (on the XY plane) of the curve at height level 'k' is called a **level curve** with constant 'k'.

• A set of some level curves of $f(x, y)$ is called a **contour map** (or contour plot) of $f(x, y)$.

Example 14.1.4 Consider $f(x, y) = 4x^2 + y^2$.

- (a) Describe level curves of $f(x, y)$
- (b) Sketch a contour map of $f(x, y)$ using level curves with $k = 0, 1, 2$.
- (c) Find the level curve which passes through the point $(1, 0)$.

Solution Done in class

Functions of three variables

Definition

Let $D \subseteq \mathbb{R}^3$. A function of three variables is a rule that assigns a unique real number $f(x, y, z)$ to each point $(x, y, z) \in D$.

The set D is called domain of $f(x, y, z)$.

Example 14.1.5 Find domain of $f(x, y, z) = \sqrt{9 - x^2 - y^2 - z^2}$.

Find $f(0, \sqrt{3}, -\sqrt{2})$.

Solution Done in class

Level surfaces of $f(x, y, z)$

- Since the graph of $f(x, y, z)$ would be in 4 dimensional space, therefore, we cannot visualize it easily.
- But we can get an idea from its level surfaces.
- The graph of $f(x, y, z) = k$ is called a level surface of $f(x, y, z)$ with constant ' k '.

Example 14.1.6 Describe the level surfaces of $f(x, y, z) = x^2 + y^2 + z^2$.

Solution Done in class.

Similarly we can define
functions of n-variables for $n > 3$

Exercise

For $f(x_1, x_2, \dots, x_n) = \sum_{k=1}^n x_k$, find $f(1, 1, \dots, 1)$

Graphing $z = f(x, y)$ using Matlab

Covered in the following (available on Course WebCT page)

- Help Sheet “Surfaces with Matlab”
- Matlab Assignment # 2

End of Section 14.1