Section 11.3 *Area in polar coordinates*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. learn how to find area bounded by polar curves

Area enclosed by polar curves

First see explanation in class

Area enclosed by $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is

$$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta$$

Procedure for finding area

- Sketch the curve
- Find α, β
- Set up and evaluate integral

It is assumed that

- $\alpha \le \beta$
- either $r \ge 0$ or $r \le 0$ for $\alpha \le \theta \le \beta$

Example 11.3.1 Set up an integral for finding area of the inner loop of limacon $r = 2 + 4\cos\theta$.

Solution Done in class

Example 11.3.2 Set up an integral for finding area of four leaved rose $r = \cos 2\theta$.

Solution Done in class

Exercise 11.3.3 Find the area of one leaf (or loop) of $r = \sin 3\theta$.

Answer: $\frac{\pi}{12}$.

Area between two polar curves

See explanation in class

If $r_{out} = f(\theta)$ is outer curve and $r_{in} = g(\theta)$ is inner curve from $\theta = \alpha$ to $\theta = \beta$.

Then the area enclosed by these two curves from $\theta = \alpha$ to $\theta = \beta$ is

$$A = \int_{\alpha}^{\beta} \frac{1}{2}\left(r_{out}^2 - r_{in}^2\right) d\theta$$

Procedure for finding area

- Sketch the curve
- Find α, β (points of intersection)
- Set up and evaluate integral

It is assumed that

- $\alpha \le \beta$
- either $r \ge 0$ or $r \le 0$ for $\alpha \le \theta \le \beta$

In the examples below, see special trick for using negative angle

Example 11.3.4 Set up an integral for finding area outside $r = 3 + 2\sin\theta$ and inside $r = 2$.

Solution Done in class

Example 11.3.5 Set up an integral for finding area inside $r = 3 + 2\sin\theta$ and outside $r = 2$.

Solution Done in class

Caution about finding points of intersections of polar curves

Equating two polar curves does not always give all points of intersection.

So the best strategy to find all points of intersections is

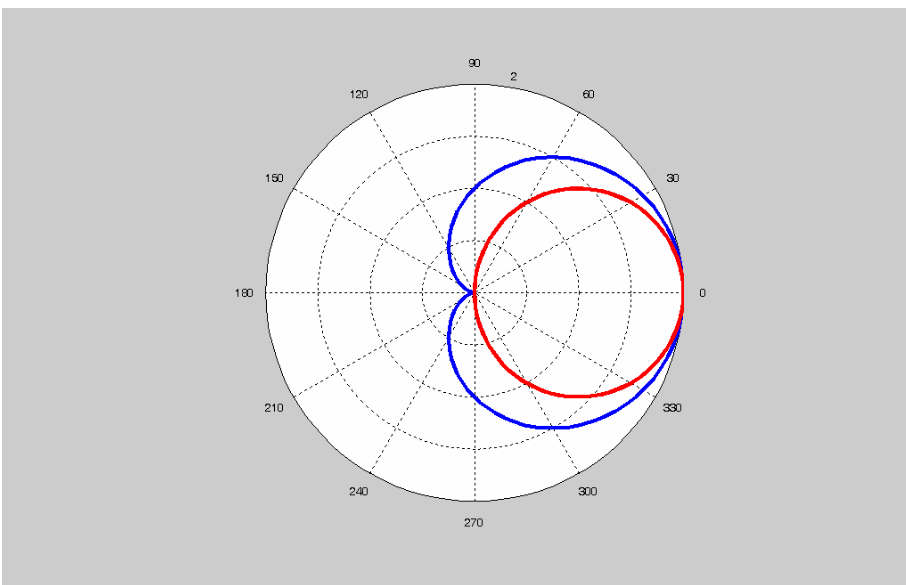
- to equate the equations and solve
- also to sketch the curve to see if there are more intersection points.

Why?
Because a point may have different representation as polar coordinates

Example 11.3.6 Find all points of intersections of $r = 1 + \cos\theta$ and $r = 2\cos\theta$ for $0 \le \theta < 2\pi$.

Solution

- By equating $1 + \cos\theta = \cos 2\theta$ we get $\cos\theta = 1 \quad \Rightarrow \quad \theta = 0$.
- From the sketch below, we see that "pole" is also a point of intersection.



End of 11.3

Do Qs: 1-30