Section 4.5 *Summary of curve sketching*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. use important information to sketch graph of a function
2. find slant asymptotes and use them in sketching graph

Main idea of this section

- Collect useful information about the function.

For example

- Asymptotes
- Increasing/decreasing intervals
- Local extrema
- Concavity intervals
- Points of inflection

We already know how to find all this information.

- Use above information to complete the graph.

See example 1 done in class

Below we see the necessary information that may be useful in graph sketching

Note

- We may not need all the pieces of information for every graph.
- You have to be clever in using it.

Useful information for graph sketching

1. Domain of function
2. Symmetries of graph
   - About Y-axis: If $x \leftrightarrow -x \Rightarrow$ no change in equation
   - About X-axis: If $y \leftrightarrow -y \Rightarrow$ no change in equation
   - About origin: If $(x, y) \leftrightarrow (-x, -y) \Rightarrow$ no change in equation
3. Intercepts
4. Asymptotes
   - Horizontal
   - Vertical
   - Slant
     [See example 7]
5. Local extrema and increasing/decreasing interval
6. Concavity and point(s) of inflection
7. Behavior of function as $x \to \infty$ or $x \to -\infty$
   - We specially look at this when there are no horizontal asymptotes.

     [See example 5]
8. Vertical tangents

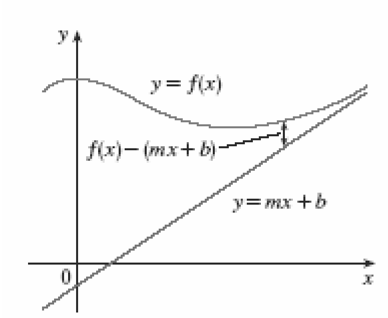
   [See example 5]

Completing the graph using above information

See examples 2, 3, 4, 5, 6 done in class

Slant (or oblique) Asymptotes

- Look at the following graph.



It runs (very close &) parallel to graph up to $x=\infty$

What's special about line y=mx+b

What happens to graph when x gets near ∞

The graph approaches (gets closer to) the slant line y=mx+b

A slant line $y = mx + b$ is called slant asymptote of graph of $f(x)$ if

- $\lim\limits_{x\to\infty}\left[f(x)-(mx+b)\right]=0$

or

- $\lim\limits_{x\to-\infty}\left[f(x)-(mx+b)\right]=0$

Special situation for rational functions

Example:

- Take $f(x)=\dfrac{x^3+2x^2+7x-3}{2x^2}$
- Using long division, we can write as

$$f(x)=\frac{1}{2}x+1+\frac{7x-3}{2x^2}$$

- Which implies

$$\lim_{x\to\pm\infty}\left[f(x)-\left(\frac{1}{2}x+1\right)\right]=\lim_{x\to\pm\infty}\frac{7x-3}{2x^2}=0$$

- Hence $y=\dfrac{1}{2}x+1$ is slant asymptote

- Given a rational function $f(x)$ with degree of numerator one more than degree of denominator.
- Using long division we can write as

$$f(x)=mx+b+g(x)$$

- So that

$$\lim_{x\to\pm\infty}\left[f(x)-(mx+b)\right]=\lim_{x\to\pm\infty}g(x)=0$$

- Hence $y=mx+b$ is slant asymptote

See example 7 done in class

End of 4.5