Section 4.4 *Indeterminate forms and l'Hospital's rule*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. use l'Hospital's rule to compute limits of indeterminate forms of the following types

    a. $\left(\frac{0}{0}\right)$, $\left(\frac{\infty}{\infty}\right)$ type

    b. $(0\cdot\infty)$, $(\infty-\infty)$ type

    c. 0^0, ∞^0, 1^∞ type

Limits of indeterminate forms of $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$ type

- Use l'Hospital's rule explained below

Called l'Hospital's Rule

If $\lim_{x \to a} \frac{f(x)}{g(x)}$ gives $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$ form

then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Also valid for

- one-sided limits
- limits $x \to \pm\infty$

See examples 1, 2, 3, 4, 5, 6 done in class

Limits of indeterminate forms of $(0 \cdot \infty)$ or $(\infty - \infty)$ type

- Reduce to previous case $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$ type
- And use l'Hospital's rule

See examples 7, 8, 9 done in class

Limits $\lim_{x\to a}\left(f(x)\right)^{g(x)}$ of the form $0^0, \infty^0, 1^\infty$

- Take 'ln' and reduce to previous cases

Procedure for finding limit

- Set $y = \left(f(x)\right)^{g(x)}$

$$\Rightarrow \lim_{x\to a} y = \lim_{x\to a}\left(f(x)\right)^{g(x)} = ?$$

- Take ln and compute $\ln\left(\lim_{x\to a} y\right)$

$$\begin{aligned}\Rightarrow \ln\left(\lim_{x\to a} y\right) &= \ln\left[\lim_{x\to a}\left(f(x)\right)^{g(x)}\right] \\ &= \lim_{x\to a}\left[\ln\left(f(x)\right)^{g(x)}\right] \\ &= \lim_{x\to a}\left[g(x)\cdot\ln\left(f(x)\right)\right] \\ &= L\end{aligned}$$

Gives $(0\cdot\infty)$ form. This can be solved as explained above

- Then $\lim_{x\to a} y = e^L$

See examples 10, 11, 12 done in class

End of 4.4