Section 4.1 *Maximum and minimum values*

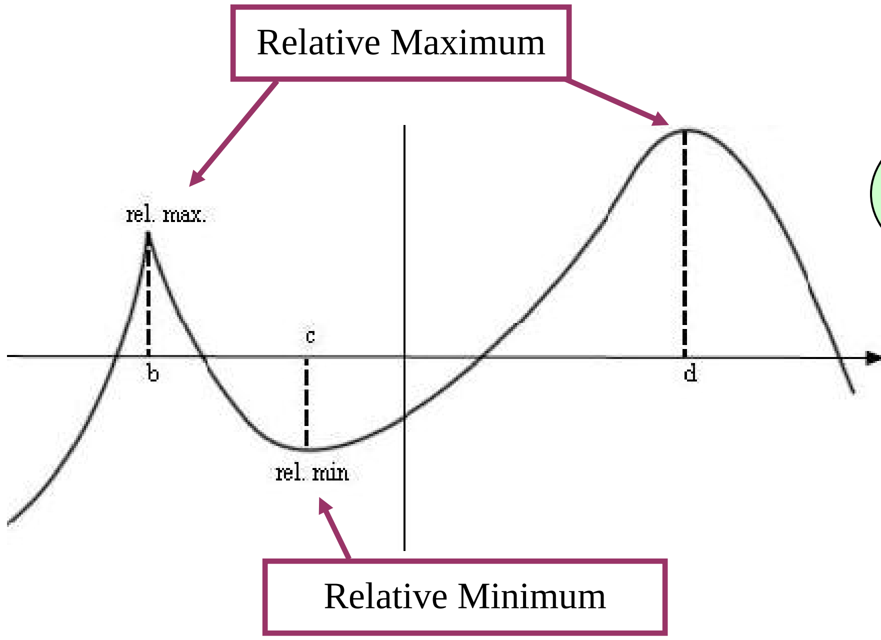
Learning outcomes

After completing this section, you will inshaAllah be able to

1. explain what is meant by relative (local) maximum and minimum
2. explain what is meant by absolute maximum and minimum
3. explain the meaning of increasing or decreasing intervals of a function
4. explain what are critical values
5. find critical values
6. determine absolute extrema of a function on a closed interval

What are relative (local) extreme points?

- Graphical explanation



See class explanation

- Given a function $f(x)$. It has

Relative maximum at x_0 if

$$f(x_0) \geq f(x) \text{ for all points x near } x_0.$$

Highest of points near x_0.

Relative minimum at x_0 if

$$f(x_0) \leq f(x) \text{ for all points x near } x_0.$$

Lowest of points near x_0.

What are absolute extreme points?

- Given a function *f(x)*. It has

Absolute maximum at x_0 if

$$f(x_0) \geq f(x) \text{ for all points x in domain.}$$

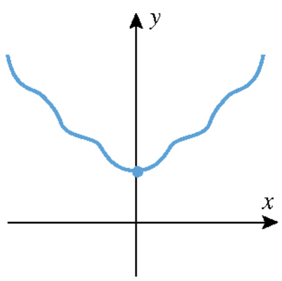
Highest of all points in domain

Absolute minimum at x_0 if

$$f(x_0) \leq f(x) \text{ for all points x in domain.}$$

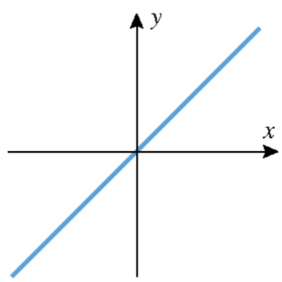
Lowest of all points in domain

- Some examples of absolute extreme points



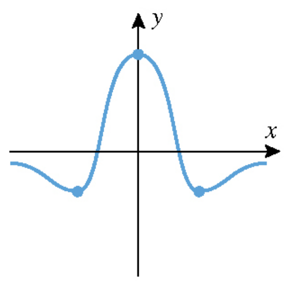
f has an absolute minimum but no absolute maximum on $(-\infty, +\infty)$.

(*a*)



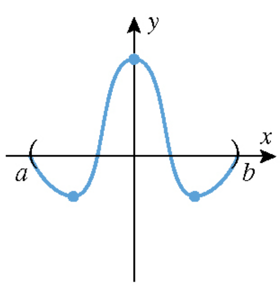
f has no absolute extrema on $(-\infty, +\infty)$.

(*b*)



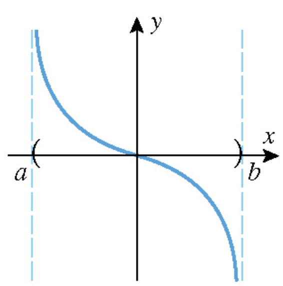
f has an absolute maximum and minimum on $(-\infty, +\infty)$.

(*c*)



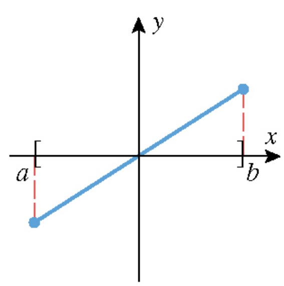
f has an absolute maximum and minimum on (a, b).

(*d*)



f has no absolute extrema on (a, b).

(*e*)



f has an absolute maximum and minimum on $[a, b]$.

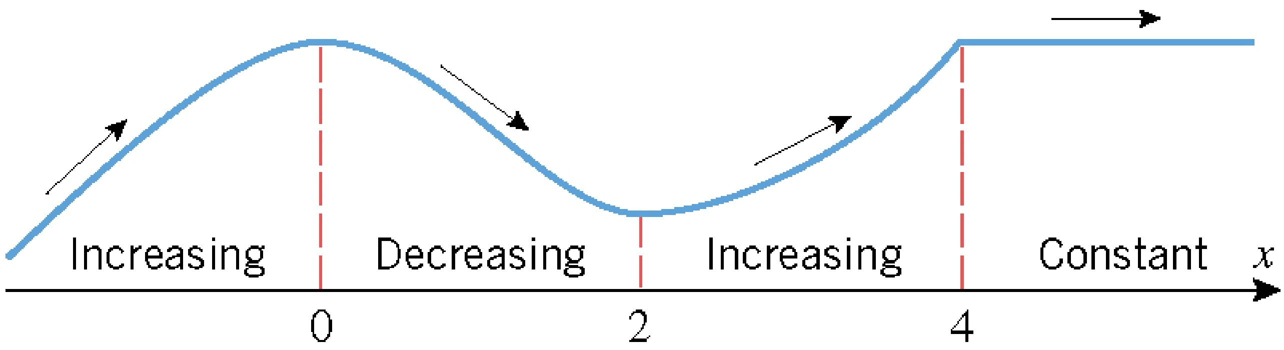
(*f*)

In general, a function may not have absolute maximum or absolute minimum or both.

See example 1 done in class

Increasing or decreasing functions

- Graphical explanation.



- Given a function $f(x)$. Then

$f' > 0$ on I $\Rightarrow$ f is increasing on I

$f' < 0$ on I $\Rightarrow$ f is decreasing on I

$f' = 0$ on I $\Rightarrow$ f is constant on I

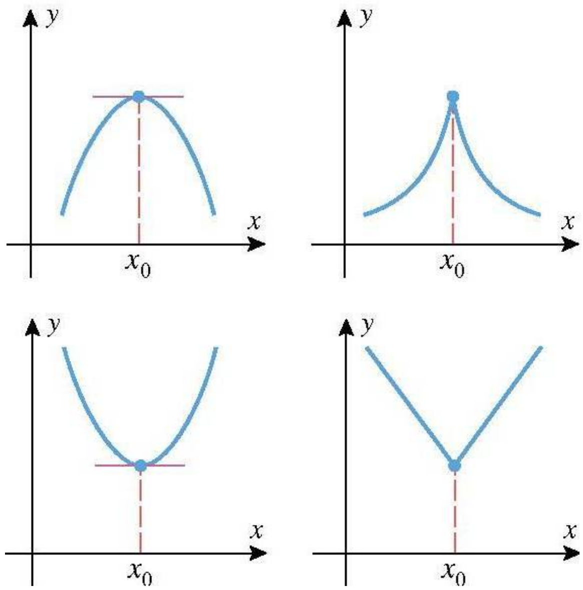
See class explanation

- Next we look at two important questions about increasing/decreasing functions.
- These questions will lead us to understand the concept of critical values.

Two important questions regarding increasing or decreasing intervals

Q.1. At which points can a function change from increasing to decreasing or decreasing to increasing?

Obviously at points where f' changes sign. Why?



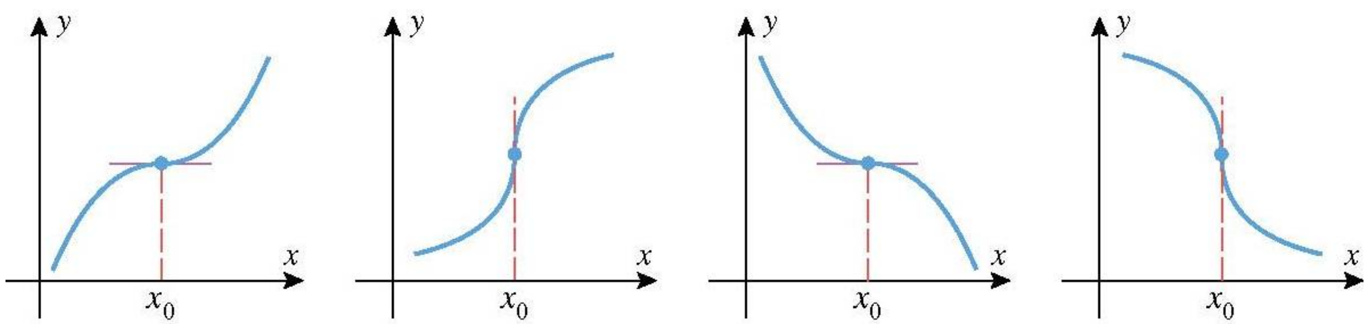
A function **can** change from increasing to decreasing or decreasing to increasing ONLY at points where $f' = 0$ or f' undefined.

- See class explanation

Q.2. At such points will the function surely change from increasing to decreasing or decreasing to increasing?

No. It will not happen always.

- See class explanation and the graphical explanation below.



How is the concept of increasing/decreasing functions related to the concept of relative extreme points?

- We can also look at relative extreme points from the following view.

Relative maximum point: where f changes from increasing to decreasing

Relative minimum point: where f changes from decreasing to increasing

- Before we learn how to find relative extrema, recall from previous page
  - Relative extrema can occur only at points where

    $f' = 0$ or f' undefined
  - But it is not necessary that all such points will be relative extreme points. We must check further to see if

    "f changes from increasing to decreasing or

    decreasing to increasing at these points".

Critical values

- Given a function $f(x)$

The values of x where

$f' = 0$ or f' undefined

are called critical values of $f(x)$

- These are possible relative extreme points
- Relative extrema can only occur at these points
- But we must check further

See example 2, 3, 4 done in class

Absolute extrema on a closed interval

- Recall from 4.1_3 that in general a function may not have absolute extrema. But for closed interval we have a definite answer.

Extreme value theorem

A continuous function has an absolute maximum and an absolute minimum on a closed interval

See graphical explanation in class

To find absolute extrema of f(x) on [a,b]

- Find critical values of *f(x)* in the interval
- Evaluate *f(x)* at all critical values
- Evaluate *f(x)* at end points of interval
- The largest value (of above) is absolute maximum
  The smallest value (of above) is absolute minimum

See class explanation

See examples 5, 6 done in class

End of 4.1