Section 3.11 *Linear approximations and differentials*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. find linear approximation of non-linear functions
2. find differential of a function
3. use differentials to approximate small changes or errors

Linear approximation

- Why do we need linear approximations?
  - See class explanation
- How to approximate?
  - Given a function $y = f(x)$.
  - Equation of its tangent line at $(a, f(a))$ is

$$y - f(a) = f'(a)(x - a)$$

or

$$y = f(a) + f'(a)(x - a)$$

- Main idea of linear approximation

  "Near point $x=a$, the tangent line and the function f(x) have approximately same graph"

  See class explanation

- **Linear Approximation of *f(x)***

  For values of x near $x=a$

  $$f(x) \approx f(a) + f'(a)(x - a).$$

See examples 1, 2 done in class

Differential of a function

Given a function $y = f(x)$.

The differential 'dy' of y is given by

$$dy = f'(x)dx$$

where dx denotes change in x

See class explanation

See example 3 done in class

Finding change Δy in $y = f(x)$ corresponding to change $\Delta x = dx$ in x

Given a function $y = f(x)$.

If x changes from x to $x+dx$ then

$$\Delta y = f(x + \Delta x) - f(x) \quad (*)$$

See class explanation to see difference between dy and Δy

See example 4 done in class

Using differentials to approximate small change Δy in the function

- Note: For small $\Delta x = dx$ we have $\Delta y \approx dy$.
- Since finding dy is easy, it is a good idea to use dy to approximately find Δy.

See example 5 done in class

Application of differentials in estimating the errors

When you make measurements "are you always exact?"

- Suppose we make a small error Δx in measuring x.
- This will obviously lead to an error Δy in $y = f(x)$.
- As seen above, for a small change Δx, we have $\Delta y \approx dy$.

So we can use dy to estimate error Δy in $y = f(x)$

We will find following types of errors

- dy: Error in y
- $\frac{dy}{y}$: Relative Error in y
- Relative error expressed as percentage: Percentage error

See class explanation

Which of these give better information?

See example 6 done in class

End of 3.11