Section 2.7 *Tangents, velocities and other rates of change*

Learning outcomes

After completing this section, you will inshaAllah be able to

1. use a special type of limit to find slopes of tangent lines
2. use a special type of limit to find rate of change of a function

Slope of tangent line to a curve y=f(x)

- Given a curve $y = f(x)$

The slope of tangent line at $(a, f(a))$ is

given by

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

See class explanation

See example 1 done in class

If we take $x = a + h$ then the above definition becomes

The slope of tangent line at $(a, f(a))$ is

given by

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

See class explanation

See example 2 done in class

Instantaneous rate of change

- Given a function $y = f(x)$

Average rate of change of $f(x)$ over $[a, a+h]$

$$y_{av} = \frac{\text{change in } f(x)}{\text{change in } x} = \frac{f(a+h) - f(a)}{h}$$

Instantaneous rate of change in $f(x)$ at $x = a$

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

See class explanation

Every day example

$s(t)$: position of object at time t

Average velocity v_{av} over interval $[t, t+h]$

= average rate of change of displacement over interval

$$\Rightarrow \quad v_{av} = \frac{s(t+h) - s(t)}{h}$$

See example 3 done in class

Velocity at time t =

Instantaneous rate of change of displacement at time t

$$\Rightarrow \quad v = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

Final remarks

- Given a function $y = f(x)$.
- We have seen in this section that limits of the form

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

are very special and can be used to answer important questions.

- From the next lecture we will focus on studying such type of limits.

End of 2.7