1. If the line $y = \alpha x + \beta$ is the slant asymptote to the curve $y = \dfrac{6x^3 - 4x^2 + 15x + 4}{2x^2 + 5}$, then $\alpha + \beta =$

(a) 1

(b) 0

(c) 2

(d) -2

(e) -1

2. The graph of $f(x) = \dfrac{1}{2}x - \sin x, \quad 0 < x < 3\pi$ is concave upward on the interval(s)

(a) $(0, \pi)$ and on $(2\pi, 3\pi)$

(b) $\left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$

(c) $\left(0, \dfrac{\pi}{2}\right)$ and on $\left(\pi, \dfrac{3\pi}{2}\right)$

(d) $\left(0, \dfrac{\pi}{2}\right)$ and on $(\pi, 3\pi)$

(e) $\left(\dfrac{3\pi}{2}, 3\pi\right)$

3. A particle moves in a straight line and has acceleration given by $a(t) = 2\sinh t$. Its initial velocity $v(0) = -\dfrac{1}{3}$ cm/s and its initial displacement is $s(0) = 0$, then $s(1) =$

(a) $\left(2\sinh 1 - \dfrac{7}{3}\right)$ cm

(b) $\left(2\cosh 1 + \dfrac{2}{3}\right)$ cm

(c) $\left(2\sinh t - \dfrac{2}{3}\right)$ cm

(d) $\left(2\cosh 1 - \dfrac{2}{3}\right)$ cm

(e) $\left(2\sinh 1 - \dfrac{5}{3}\right)$ cm

4. The asymptotes of $f(x) = \dfrac{x^3 + 2x^2 - 3x}{2x^3 - x^2 - x}$ are

(a) one horizontal and one vertical asymptotes

(b) one horizontal and two vertical asymptotes

(c) no horizontal and three vertical asymptotes

(d) one horizontal and three vertical asymptotes

(e) one horizontal, one slant, and one vertical asymptotes

5. The radius of a circle increases from 3 cm to 3.025 cm. Using differentials, the best approximation in the increase of its area is equal to

(a) $0.15\,\pi$ cm^2

(b) $0.75\,\pi$ cm^2

(c) $0.45\,\pi$ cm^2

(d) $0.09\,\pi$ cm^2

(e) $0.18\,\pi$ cm^2

6. The graph of the function

$$f(x) = (x-3)(x+1)^3$$

is increasing on

(a) $(2, \infty)$

(b) $(-\infty, -1)$ and on $(2, \infty)$

(c) $(-\infty, \infty)$

(d) $(-\infty, -2)$ and on $(1, \infty)$

(e) $(-\infty, -1)$ and on $(3, \infty)$

7. If $\lim_{h \to 0} \dfrac{g(2+h) - g(2)}{h} = 5$ and $g(2) = -3$, then the y-intercept of the tangent line to the graph of g at $(2, -3)$ is

(a) $(0, -13)$

(b) $(0, 11)$

(c) $(0, -11)$

(d) $(0, 9)$

(e) $(0, -15)$

8. If $f(x) = 4^{\sin(\pi x)}$, then $f'\left(\dfrac{1}{6}\right) =$

(a) $\pi\sqrt{3}\ln 4$

(b) $\pi \ln 2$

(c) $-2\pi\sqrt{3}\ln 4$

(d) $3\pi\sqrt{3}\ln 4$

(e) $\pi \ln 4$

9. The sum of all critical points of the function $f(x) = \dfrac{x^2+1}{\sqrt{2x+1}}$ is

(a) $\dfrac{1}{3}$

(b) $-\dfrac{1}{2}$

(c) $-\dfrac{5}{6}$

(d) $\dfrac{1}{6}$

(e) $\dfrac{4}{3}$

10. Using Newton's Method to estimate $\sqrt[5]{3}$ with $x_1 = 1,$ we find that $x_2 =$

(a) 1.4

(b) 1.5

(c) 1.6

(d) 1.2

(e) 1.8

11. The volume of a right circular cylinder is decreasing at the rate of 88π cm^3/s, while the height is increasing at the rate of 2 cm/s. Then at the instant when the radius is 2 cm and the height is 6 cm, the radius is [Volume of a cylinder = Area of base $\times$ height].

(a) decreasing at the rate of 4 cm/s

(b) increasing at the rate of 2 cm/s

(c) decreasing at the rate of 11 cm/s

(d) increasing at the rate of $\frac{1}{2}$ cm/s

(e) decreasing at the rate of $\frac{2}{3}$ cm/s

12. The limit $\lim_{x\to 0^+} [(\sin 2x)(\ln 3x)]$

(a) is equal to 0

(b) is equal to $-\frac{3}{2}$

(c) is equal to $-\frac{2}{3}$

(d) is equal to -6

(e) does not exist

13. Which one of the following statements is **TRUE** for any given function $f(x)$?

(a) If $f''(x)$ exists at $x = a$, then $f'(x)$ is continuous at $x = a$

(b) If $\lim_{x \to a} f(x)$ exists, then $f(x)$ is continuous at $x = a$

(c) If $\lim_{x \to a} f(x)$ exists, then $f(x)$ is defined at a

(d) If $\lim_{x \to a} f(x) = f(a)$, then $f'(x)$ exists at $x = a$

(e) If $f'(x)$ exists at $x = a$, then $f''(x)$ exists at $x = a$

14. If $f(x) = \operatorname{sech}\left(\frac{x}{2}\right)$, then $f'(\ln 4) =$

(a) $-\frac{6}{25}$

(b) $\frac{12}{25}$

(c) $-\frac{3}{25}$

(d) $\frac{16}{25}$

(e) $-\frac{4}{25}$

15. The number of points that satisfy the conclusion of the Rolle's Theorem for the function $f(x) = x^4 - 4x^2 + 3$ on the interval $[-1, 1]$ is

(a) 1

(b) 0

(c) 2

(d) 3

(e) 4

16. If M_{max} and N_{min} are, respectively, the numbers of the local maximum values and the local minimum values of the function $f(x) = x^{4/5}(x-4)^2$, then

(a) $M_{\text{max}} = 1$ and $N_{\text{min}} = 2$

(b) $M_{\text{max}} = 2$ and $N_{\text{min}} = 1$

(c) $M_{\text{max}} = 1$ and $N_{\text{min}} = 1$

(d) $M_{\text{max}} = 0$ and $N_{\text{min}} = 2$

(e) $M_{\text{max}} = 2$ and $N_{\text{min}} = 0$

17. The graph of the function $f(x) = xe^{1-2x}$ has

(a) only one inflection point $\left(1, \frac{1}{e}\right)$

(b) no inflection points

(c) only one inflection point $\left(\frac{1}{2}, 1\right)$

(d) two inflection points $\left(\frac{1}{2}, 1\right)$ and $\left(1, \frac{1}{e}\right)$

(e) two inflection points $(0, e)$ and $\left(1, \frac{1}{e}\right)$

18. If $f'(x) = \dfrac{2x^4 - 3\sqrt{x}}{x}$ and $f(1) = \dfrac{1}{2}$, then $f(x) =$

(a) $\frac{1}{2}x^4 - 6\sqrt{x} + 6$

(b) $\frac{1}{4}x^4 - 3\sqrt{x} + \frac{13}{4}$

(c) $\frac{2}{5}x^3 - 3\ln|x| + \frac{1}{10}$

(d) $2x^4 + 6\sqrt{x} - \frac{15}{2}$

(e) $\frac{1}{2}x^4 - 6\sqrt{x}$

19. If A is the area of the largest rectangle that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 27 - x^2$, then $A =$

(a) 108

(b) 95

(c) 64

(d) 116

(e) 81

20. Given $f(x) = \begin{cases} 2 & \text{if } x < -2 \\ |x| & \text{if } -2 \leq x < 1 \\ \sqrt{x-1} & \text{if } x \geq 1 \end{cases}$, which one of the following statements is **FALSE** about f? [Hint: Sketch the graph of f]

(a) f has a removable discontinuity at $x = 1$

(b) f is continuous at $x = -2$

(c) f is decreasing on $(-2, 0)$

(d) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

(e) $\lim_{x \to 1} f(x)$ does not exist

21. If L is the linearization of $f(x) = \sin^{-1} x$ at $x = \frac{1}{2}$, then $L\left(\frac{1}{3}\right) =$

(a) $\frac{\pi}{6} - \frac{\sqrt{3}}{9}$

(b) $\frac{\pi}{3} - \frac{\sqrt{3}}{6}$

(c) $\frac{\pi}{6} - \frac{\sqrt{3}}{3}$

(d) $\frac{\pi}{3} - \frac{\sqrt{3}}{3}$

(e) $\frac{\pi}{6} - \frac{2\sqrt{3}}{3}$

22. The slope of the tangent line to the graph of $y\sqrt{x} - x\sqrt{y} - 12 = 0$ at the point $(9, 16)$ is equal to

(a) $\frac{32}{45}$

(b) $\frac{28}{15}$

(c) $\frac{32}{9}$

(d) $\frac{14}{25}$

(e) $\frac{32}{3}$

23. Suppose that f is continuous on $[6, 15]$ and differentiable on $(6, 15)$. If $f(6) = -2$, and $f'(x) \leq 10$ for $6 < x < 15$, then the largest possible value of $f(15)$ is

(a) 88

(b) 10

(c) -10

(d) 90

(e) 9

24. Given that $f(x) = \dfrac{2x}{\sqrt{x^2 - 4}}$ and $f'(x) = \dfrac{-8}{(x^2 - 4)^{3/2}}$, which one of the following statements is **TRUE** about the graph of f?

(a) The graph has no inflection points

(b) The graph has only one vertical asymptote

(c) The graph has only one local minimum

(d) The graph is concave downward on $(2, \infty)$

(e) The graph has no horizontal asymptotes

25. Given $f(x) = 1 + (x+1)^2$ where $-2 \leq x < 5$, which one of the following statements is **TRUE**? [Hint: Sketch the graph of f]

(a) f has no absolute or local maximum

(b) f has no absolute or local minimum

(c) f has local minimum but no absolute minimum

(d) f has local and absolute minimum $f(0) = 2$

(e) f has absolute maximum $f(5) = 37$

26. The limit $\lim\limits_{x \to \infty} (\sqrt{4x^2 + 3x} - 2x)$

(a) is equal to $\dfrac{3}{4}$

(b) is equal to $\dfrac{3}{8}$

(c) is equal to $\dfrac{2}{3}$

(d) is equal to 0

(e) does not exist

27. If (α, β) is the point on the curve $y = 1 + 30x^2 - 5x^3$ at which the tangent line has the largest slope, then $\alpha + \beta =$

(a) 83

(b) 72

(c) 86

(d) 77

(e) 80

28. Which one of the following statements is **TRUE** about the function $f(x) = \frac{3}{2}(x-1)^{2/3} + 8$?

(a) f has a vertical tangent line at $x = 1$

(b) f has a vertical asymptote at $x = 1$

(c) f is discontinuous at $x = 1$

(d) f is differentiable at $x = 1$

(e) f has no critical numbers