King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
**Math 102.07 Quiz V Fall (051)**

ID #: ______________ NAME: Key Solution

Serial # ______________ Section #: ______________

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1. Evaluate $\int_0^4 x^3\sqrt{16-x^2}\,dx$. Let $x = 4\sin\theta \Rightarrow dx = 4\cos\theta\, d\theta$

$x=4 \Rightarrow \theta = \frac{\pi}{2}$, $x=0 \Rightarrow \theta = 0$.

$$\int_0^4 x^3\sqrt{16-x^2}\,dx = \int_0^{\frac{\pi}{2}} (4^3\sin^3\theta)\sqrt{16-16\sin^2\theta}(4\cos\theta\, d\theta)$$

$$= 4^5\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^2\theta\, d\theta = 4^5\int_0^{\frac{\pi}{2}} \sin^2\theta\sin\theta\cos^2\theta\, d\theta$$

$$= 4^5\int_0^{\frac{\pi}{2}} \sin\theta(1-\cos^2\theta)\cos^2\theta\, d\theta =$$

$$4^5\int_0^{\frac{\pi}{2}} (\cos^2\theta - \cos^4\theta)\sin\theta\, d\theta = 1024\left|-\frac{\cos^3\theta}{3} + \frac{\cos^5\theta}{5}\right|_0^{\frac{\pi}{2}} = 1024\left[(0-0)-\left(-\frac{1}{3}+\frac{1}{5}\right)\right] = \frac{2048}{15}$$

Another method: let $u = 16-x^2$.

2. Compute $\int \frac{dx}{x^3+x}$. $\frac{1}{x^3+x} = \frac{1}{x(x^2+1)} = \frac{A_1}{x} + \frac{A_2x+B_2}{x^2+1}$

$\Rightarrow 1 = A_1(x^2+1) + (A_2x+B)x$

We set $x=0 \Rightarrow \boxed{A_1=1}$. Setting $A_1=1$ in the equation we get $1 = x^2+1+(A_2x+B_2)x$

$\Rightarrow -x^2 = (A_2x+B_2)x \Rightarrow A_2x+B_2 = -x$

Setting $x=0$ we get $\boxed{B_2=0}$ and $A_2x=-x$

$\Rightarrow \boxed{A_2=-1}$. Hence $\frac{1}{x^3+x} = \frac{1}{x} + \frac{-x}{x^2+1}$

$$\int\frac{dx}{x^3+x} = \int\left(\frac{1}{x} - \frac{x}{x^2+1}\right)dx = \ln|x| - \frac{1}{2}\ln|x^2+1| + C$$

$$= \ln\left|\frac{x}{\sqrt{x^2+1}}\right| + C$$

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**Math 102.10** **Quiz VI** **Fall (051)**

ID #: ____________________ NAME: Key Solution

Serial # ____________________ Section #: ____________________

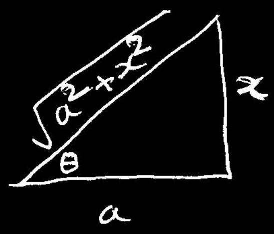
1. Find $\int \frac{dx}{(a^2+x^2)^{3/2}}$. $a > 0$

Let $x = a\tan\theta \Rightarrow dx = a\sec^2\theta\, d\theta$

and $(a^2+x^2)^{\frac{3}{2}} = (a^2+a^2\tan^2\theta)^{\frac{3}{2}} = a^3(\sec^3\theta)$

$$\int \frac{dx}{(a^2+x^2)^{\frac{3}{2}}} = \int \frac{a\sec^2\theta\, d\theta}{a^3\sec^3\theta} = \frac{1}{a^2}\int \frac{d\theta}{\sec\theta}$$

$$= \frac{1}{a^2}\int \cos\theta\, d\theta = \frac{\sin\theta}{a^2} + C$$




$$= \frac{1}{a^2}\, \frac{x}{\sqrt{a^2+x^2}} + C$$

2. Compute $\int \frac{dx}{x(x-1)^2}$. $\frac{1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$

$\Rightarrow 1 = A(x-1)^2 + Bx(x-1) + Cx$

Setting $x=0 \Rightarrow \boxed{A=1}$. Setting $x=1 \Rightarrow \boxed{C=1}$.

Substituting in the equation by $A = C = 1$

we get $(x-1)^2 + Bx(x-1) + x = 1$

Setting $x=2 \Rightarrow 1 = 1 + 2B + 2 \Rightarrow \boxed{B=-1}$ Therefore

$$\int \frac{dx}{x(x-1)^2} = \int \left(\frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2}\right) dx =$$

$$\ln|x| - \ln|x-1| - \frac{1}{x-1} + C$$

$$= \ln\left|\frac{x}{x-1}\right| - \frac{1}{x-1} + C.$$

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**Math 102.10 Quiz V Fall (051)**

ID #: ____________ NAME: Key Solution

Serial # ____________ Section #: ____________

1. Calculate $\int_0^1 x^3 e^{x^2}\, dx$. Using integration by parts the following are choices for $u$: $x^3$, $e^{x^2}$, $xe^{x^2}$, $x^2$, $x^2e^{x^2}$, $x$ and $x^3e^{x^2}$. The only choice that works is $u = x^2$, $dv = xe^{x^2}$. Thus $du = 2x\,dx$ and $v = \frac{1}{2}e^{x^2}$

$$\int_0^1 x^3 e^{x^2} dx = \int x^2 \cdot x e^{x^2} dx = x^2 \frac{e^{x^2}}{2}\Big|_0^1 - \int_0^1 x e^{x^2} dx$$

$$= \frac{e}{2} - \frac{1}{2} e^{x^2}\Big|_0^1 = \frac{e}{2} - \left[\frac{e}{2} - \frac{1}{2}\right] = \frac{1}{2}$$

OR: $I = \int_0^1 x^3 e^{x^2} dx$. Let $u = x^2 \Rightarrow I = \frac{1}{2}\int_0^1 u e^u du$ and then use integration by parts.

2. Find $\int \tan^5 x\, dx$. $= \int \tan^3 x \tan^2 x\, dx =$

$$\int \tan^3 x(\sec^2 x - 1)\, dx = \int \tan^3 x \sec^2 x\, dx - \int \tan^3 x\, dx$$

$$= \int \tan^3 x \sec^2 x\, dx - \int \tan x\,(\sec^2 x - 1)\, dx$$

$$= \int \tan^3 x \sec^2 x\, dx - \int \tan x \sec^2 x\, dx - \int \tan x\, dx$$

$$= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} - \ln|\cos x| + C$$

OR $\int \tan^5 x\, dx = \int (\tan^4 x) \tan x\, dx = \int (\sec^2 - 1)^2 \tan x\, dx$

$$= \int (\sec^4 x \tan x - 2\sec^2 x \tan x + \tan x)\, dx = \frac{\sec^4 x}{4} - \tan^2 x + \ln|\sec x| + C$$

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**Math 102.07 Quiz VI Fall (051)**

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1. Calculate $\int \sin^{-1} x \, dx$. Solution: There is no other choice but to set $\sin^{-1} x = u$ and $dv = dx$. Then $du = \frac{dx}{\sqrt{1-x^2}}$ and $v = x$ and

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx$$

$$= x \sin^{-1} x + \frac{1}{2} \frac{\sqrt{1-x^2}}{\frac{1}{2}} + C$$

$$= x \sin^{-1} x + \sqrt{1-x^2} + C.$$

2. $\int \tan^3 x \sec^5 x \, dx.$ $= \int \tan^2 x \sec^4 x \sec x \tan x \, dx$

$$= \int (\sec^2 x - 1) \sec^4 x \cdot \sec x \tan x \, dx$$

$$= \int (\sec^6 x - \sec^4 x) \sec x \tan x \, dx$$

$$= \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$$