Key Soluti

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Math 102.14, Quiz-IV, Spring 2006

1. $\int \frac{3x\,dx}{(x-1)(x+2)}$

$$\frac{3x}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2)+B(x-1)}{(x-1)(x+2)} =$$

$$3x = A(x+2) + B(x-1)$$

Let $x=1 \Rightarrow 3 = 3A \Rightarrow \boxed{A=1}$

Let $x=-2 \Rightarrow -6 = -3B \Rightarrow \boxed{B=2}$

$$\int \frac{3x\,dx}{(x-1)(x+2)} = \int \left(\frac{1}{x-1} + \frac{2}{x+2}\right) dx$$

$$= \ln|x-1| + 2\ln|x+2| + C$$

$$= \ln|x-1|(x+2)^2 + C$$

2. $\int_0^{\frac{3}{\sqrt{2}}} \frac{x^2\,dx}{(9-x^2)^{\frac{3}{2}}}$ We let $x = 3\sin\theta \Rightarrow dx = 3\cos\theta\,d$

$x = \frac{3}{\sqrt{2}} \Rightarrow \frac{3}{\sqrt{2}} = 3\sin\theta \Rightarrow \theta =$

$$\int_0^{\frac{3}{\sqrt{2}}} \frac{x^2\,dx}{(9-x^2)^{\frac{3}{2}}} = \int_0^{\frac{\pi}{4}} \frac{\sin^2\theta\,(3\cos\theta\,d\theta)}{(9-9\sin^2\theta)^{\frac{3}{2}}} \quad x=0 \Rightarrow \theta = 0$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin^2\theta\cos\theta}{27\cos^3\theta}\,d\theta = \int_0^{\frac{\pi}{4}} \frac{\sin^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\frac{\pi}{4}} \tan^2\theta\,d$$

$$= \int_0^{\frac{\pi}{4}} [\sec^2\theta - 1]\,d\theta = \Big[\tan\theta - \theta\Big]_0^{\frac{\pi}{4}} = \left[\left(1-\frac{\pi}{4}\right) - (0-\right.$$

$$= 1 - \frac{\pi}{4}$$