

Key Solution

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$$\begin{aligned} 1. \int \sec^3 x \tan^3 x \, dx &= \int \sec^2 \theta \tan^2 \theta (\sec \theta \tan \theta) \, d\theta \\ &= \int \sec^2 \theta (\sec^2 \theta - 1) (\sec \theta \tan \theta) \, d\theta \\ &= \int [\sec^4 \theta - \sec^2 \theta] \sec \theta \tan \theta \, d\theta \\ \text{We let } u &= \sec \theta \Rightarrow du = \sec \theta \tan \theta \, d\theta \Rightarrow \\ \int \sec^3 x \tan^3 x \, dx &= \int [u^4 - u^2] \, du = \frac{u^5}{5} - \frac{u^3}{3} + C \\ &= \frac{\sec^5 \theta}{5} - \frac{\sec^3 \theta}{3} + C \end{aligned}$$

$$\begin{aligned} 2. \int x^3 e^{x^2} \, dx \quad \text{let } u &= x^2 \quad dv = x e^{x^2} \, dx \Rightarrow \\ du &= 2x \, dx \quad v = \frac{1}{2} e^{x^2} \\ \int x^3 e^{x^2} \, dx &= \frac{1}{2} x^2 e^{x^2} - \int \frac{1}{2} (2x) e^{x^2} \, dx \\ &= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} \int 2x e^{x^2} \, dx = \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C \\ &= \frac{1}{2} e^{x^2} [x^2 - 1] + C \end{aligned}$$