Key Solution

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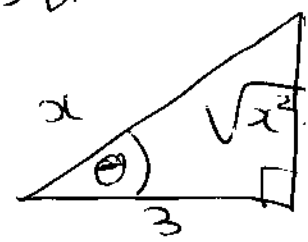
Math 102.09, Quiz-IV, Spring 2006

1. $\int \frac{\sqrt{x^2-9}}{x}\,dx$ We let $x = 3\sec\theta \Rightarrow dx = 3\sec\theta \tan$

$$= \int \frac{\sqrt{9\sec^2\theta - 9}}{3\sec\theta}\, 3\sec\theta\tan\theta\, d\theta = \int \frac{(3\tan\theta)(3\sec\theta\tan\theta)}{3\sec\theta}$$

$$= 3\int \tan^2\theta\, d\theta = 3\int (\sec^2\theta - 1)\, d\theta$$

$$= 3\tan\theta - 3\theta + C$$



$$= 3\,\frac{\sqrt{x^2-9}}{3} - 3\sec^{-1}\frac{x}{3} + C$$

$$= \sqrt{x^2-9} - 3\sec^{-1}\frac{x}{3} + C$$

$x = 3\sec\theta$

2. $\int x^2 e^{-2x}\,dx$ We let $u = x^2$, $dv = e^{-2x}\,dx \Rightarrow$

$du = 2x\,dx$ and $v = \frac{-e^{-2x}}{2}$. $\Rightarrow$

$$\int x^2 e^{-2x}\,dx = -\frac{1}{2}x^2 e^{-2x} - \int 2x\left(\frac{-e^{-2x}}{2}\right)dx$$

$$= -\frac{1}{2}x^2 e^{-2x} + \int x e^{-2x}\,dx$$

$I_1 = \int x e^{-2x}\,dx$. We let $u = x$ and $dv = e^{-2x}$

$\Rightarrow du = dx$ and $v = \frac{-e^{-2x}}{2}$

$$I_1 = -\frac{1}{2}x e^{-2x} + \int \frac{1}{2}e^{-2x}\,dx = -\frac{1}{2}x e^{-2x} - \frac{1}{2}\left(\frac{1}{2}e^{-2x}\right.$$

$$\int x^2 e^{-2x}\,dx = -\frac{1}{2}x^2 e^{-2x} - \frac{1}{2}x e^{-2x} - \frac{1}{4}e^{-2x} + C$$

$$= -\frac{1}{2}e^{-2x}\left(x^2 + x + \frac{1}{2}\right) + C$$