

Key Solution

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Math 102.09, Quiz-III, Spring 2006

1. $\int 42 \cos x \sin x (\sin x + 1)^5 dx$ We let $u = 1 + \sin x \Rightarrow \sin x = u - 1$
 $du = \cos x dx$

$$= 42 \int (u-1) u^5 du = 42 \int (u^6 - u^5) du =$$

$$42 \left[\frac{u^7}{7} - \frac{u^6}{6} \right] + C = 6u^7 - 7u^6 + C$$

$$= 6(1 + \sin x)^7 - 7(1 + \sin x)^6 + C$$

$$= (1 + \sin x)^6 \left[-7 + 6(1 + \sin x) \right] + C$$

$$= (1 + \sin x)^6 (-1 + \sin x) + C$$

2. $\int \frac{2x dx}{(x+1)(x^2+1)}$

$$\frac{2x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Cx+D}{x^2+1} = \frac{A(x^2+1) + (x+1)(Cx+D)}{(x+1)(x^2+1)}$$

$$\Rightarrow 2x = A(x^2+1) + (x+1)(Cx+D)$$

let $x=0 \Rightarrow 0 = A+D$

let $x=-1 \Rightarrow -2 = 2A \Rightarrow \boxed{A=-1}$

let $x=1 \Rightarrow 2 = 2A + 2(C+D) \Rightarrow 2 = -2 + 2C + 2D$

$\Rightarrow \boxed{C=1}$

$$\int \frac{2x}{(x+1)(x^2+1)} dx = \int \left(\frac{-1}{x+1} + \frac{x+1}{x^2+1} \right) dx =$$

$$\int \left(\frac{-1}{x+1} + \frac{x}{x^2+1} + \frac{1}{x^2+1} \right) dx = -\ln|x+1| + \frac{1}{2} \ln|x^2+1|$$

$$+ \tan^{-1} x + C$$