Key Solution

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1. $\int 42\cos x\ \sin x(\sin x+1)^5\,dx$ We let $u=1+\sin x \Rightarrow \sin x = u-1$, $du=\cos x\,dx$

$= 42\int (u-1)u^5\,du = 42\int (u^6-u^5)\,du =$

$42\left[\frac{u^7}{7}-\frac{u^6}{6}\right]+C = 6u^7-7u^6+C$

$= 6(1+\sin x)^7 - 7(1+\sin x)^6 + C$

$= (1+\sin x)^6\left[-7+6(1+\sin x)\right]+C$

$= (1+\sin x)^6(-1+\sin x)+C$

2. $\int \frac{2x\,dx}{(x+1)(x^2+1)}$

$\frac{2x}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Cx+D}{x^2+1} = \frac{A(x^2+1)+(x+1)(Cx+D)}{(x+1)(x^2+1)}$

$\Rightarrow 2x = A(x^2+1)+(x+1)(Cx+D)$

Let $x=0 \Rightarrow 0 = A+D$ $\boxed{D=1}$

Let $x=-1 \Rightarrow -2 = 2A \Rightarrow \boxed{A=-1}$

Let $x=1 \Rightarrow 2 = 2A+2(C+D) \Rightarrow 2=-2+2C+2$

$\Rightarrow \boxed{C=1}$

$\int \frac{2x}{(x+1)(x^2+1)}\,dx = \int\left(\frac{-1}{x+1}+\frac{x+1}{x^2+1}\right)dx =$

$\int\left(\frac{-1}{x+1}+\frac{x}{x^2+1}+\frac{1}{x^2+1}\right)dx = -\ln|x+1|+\frac{1}{2}\ln|x^2+1|+\tan^{-1}x + C.$