King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

Math 102.07 & .08 Final Exam First Semester 2003-2004(031)

ID #: ______________________ NAME: ______________________________

Serial # ______________________ Section #: ______________

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1. Find the area of the region R bounded by the line $y = \frac{1}{2}x$ and the parabola $y^2 = 8 - x$.

2. Evaluate the integral $\int \frac{\sqrt{4x^2 - 1}}{x^4}\, dx$. [Hint Let $u = 1/x$].

3. Evaluate $\int_0^{\pi/4} \tan^6 x \, dx$.

4. Compute the arc length of the curve $y = \ln x$ over the interval $[1, 2]$.

5. Evaluate $\int \frac{x^4}{\sqrt{1 - x^{10}}}\, dx$.

6. Consider the region in the first quadrant, bounded by the curves $y^2 = x$ and $y = x^3$. Find the solid obtained by rotating this region about the y-axis.

7. Find $\int_1^{\sqrt{3}} \frac{5x^3 - 3x^2 + 2x - 1}{x^4 + x^2}\, dx$.

8. Evaluate the integral $\int_0^1 \frac{\ln x}{x^2}\, dx$ if possible.

9. Derive the Maclaurin series for $\sinh x$ and $\cosh x$. Find also the interval of convergence and radius of convergence for $\cosh x$.

10. Find $\int x^2 \sin x^{3/2} dx$. [Hint: Let $u = x^{3/2}$ first]

11. Show that the series $\sum_{n=1}^{\infty} (-1)^n \frac{\tan^{-1} n}{n}$ is conditionally convergent [i.e., convergent but not absolutely convergent].

12. Test the following series for convergence:

(a) $\sum_{n=0}^{\infty} \frac{n^2+1}{e^n (n+1)^2}$.

(b) $\sum_{n=0}^{\infty} \frac{\sqrt{n}}{\ln(n+1)}$.

(c) $\sum_{n=1}^{\infty} \frac{2+\sin n}{n^2}$.

(d) $\sum_{n=0}^{\infty} \frac{1}{2^{n+(-1)^n}}$.

13. Use power series representation to approximate the value of $\int_0^1 \frac{1-\cos x}{x^2} dx$ accurate to 2 decimal places.

14. Determine whether the sequence $\{2^{\cos n\pi}\}_{n=1}^{\infty}$ converges or diverges.