King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

Math 102.09 & 14 **Exam I** **Form B** **Second Semester (052)**

ID #: 70663 NAME: Key Solution

Serial # ______________ Section #: ______________

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1. Given $F(x) = \int_x^0 \sqrt{t^2+1}\, dt$, compute

   (a) $F(0)$. (b) $F'(0)$. (c) $F'(1)$. (d) $F'(x)$. (e) $F''(x)$.

$F(x) = \int_x^0 \sqrt{t^2+1}\, dt = -\int_0^x \sqrt{t^2+1}\, dt$

a) $F(0) = 0$ d) $F'(x) = -\sqrt{x^2+1}$ b) $F'(0) = -1$

c) $F'(1) = -\sqrt{2}$ e) $F''(x) = -\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x) = \frac{-x}{\sqrt{x^{\cdots}}}$

2. Find f given that $f''(x) = 6x - 2$, $f'(0) = 1$, $f(0) = 2$.

$f''(x) = 6x - 2 \Rightarrow f'(x) = 3x^2 - 2x + C_1$, $f'(0) = 1$

$\Rightarrow C_1 = 1 \Rightarrow f'(x) = 3x^2 - 2x + 1 \Rightarrow f(x) = x^3 - x^2 + \cdots$

$f(0) = 2 \Rightarrow f(x) = x^3 - x^2 + x + 2$

3. Evaluate $\int_1^0 \frac{dx}{\sqrt{2x - x^2}}$. This an improper integral because the integrand has an infinite discontility a[t] $x = 0$

$\int_1^0 \frac{dx}{\sqrt{2x-x^2}} = \lim_{k\to 0^+} \int_1^k \frac{dx}{\sqrt{1-(x-1)^2}} = \lim_{k\to 0^+} \left[\sin^{-1}(x-1)\right]_1^k$

$= \lim_{k\to 0^+}\left[\sin^{-1}(k-1) - \sin^{-1} 0\right] = \lim(\sin^{-1}(-1) - \sin\cdots$

$= -\frac{\pi}{2} - 0 = -\frac{\pi}{2}$.

Note: $2x - x^2 = -(x^2 - 2x) = -(x^2 - 2x + 1) + 1 = 1 - (x-1\cdots$

4. Evaluate $\int x^3(x^2-1)^7\,dx$.

$\frac{1}{2}\int x^2(x^2-1)^7(2x)\,dx$ we let $u = x^2-1 \Rightarrow du = 2x\,dx$

$x^2 = u+1$

$= \frac{1}{2}\int (u+1)u^7\,du = \frac{1}{2}\int (u^7+u^8)$

$= \frac{1}{2}\left[\frac{u^8}{8}+\frac{u^9}{9}\right]+C$

$= \frac{1}{2}\left[\frac{(x^2-1)^8}{8}+\frac{(x^2-1)^9}{9}\right]+C$

5. Sketch the region bounded by $y^2 = 2x$ and $x^2 = 3y$ and find its area.

$y^2 = 2x$, $x^2 = 3y \Rightarrow x^4 = 9y^2$

$\Downarrow$

$9y^2 = 18x \Rightarrow 18x = 9y^2 = x^4$

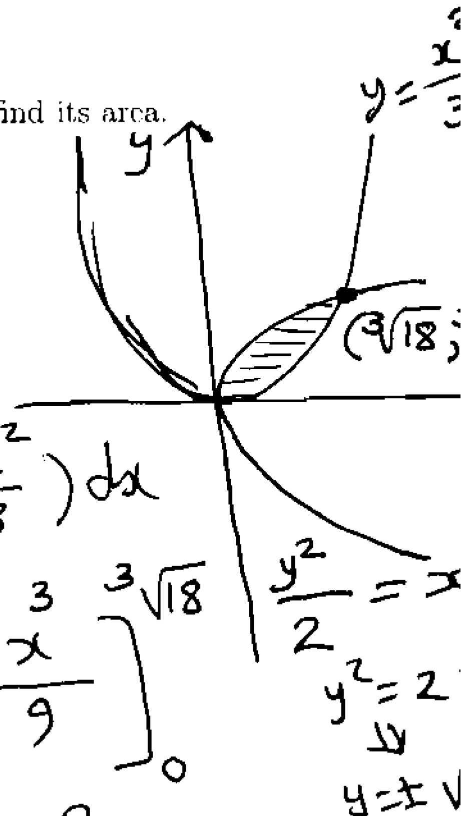
$\Rightarrow x^4 - 18x = 0 \Rightarrow x(x^3-18) = 0$

$\therefore x = 0$ or $x = \sqrt[3]{18}$

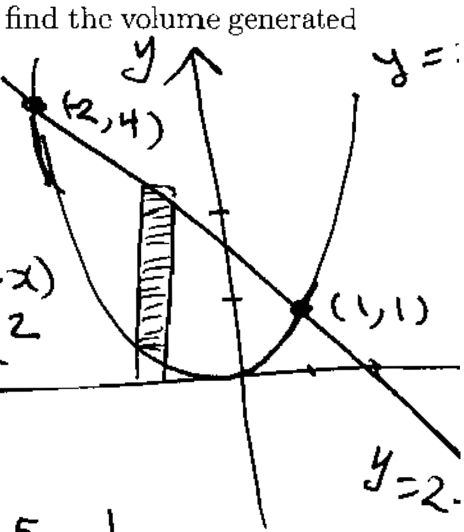
The area needed $A = \int_0^{\sqrt[3]{18}} \left(\sqrt{2x} - \frac{x^2}{3}\right)dx$

$= \int_0^{\sqrt[3]{18}} \left(\sqrt{2}\,x^{\frac{1}{2}} - \frac{x^2}{3}\right)dx = \left[\sqrt{2}\,\frac{2x^{\frac{3}{2}}}{3} - \frac{x^3}{9}\right]_0^{\sqrt[3]{18}}$

$= \frac{2}{3}\sqrt{2}\sqrt{18} - \frac{18}{9} = \frac{2}{3}(6) - 2 = 2$



6. Sketch the region Ω bounded by $y = x^2$ and $y = 2 - x$ and find the volume generated by revolving Ω about the x-axis.



$x^2 = y = 2 - x \Rightarrow x^2 + x - 2 = 0$
$\Rightarrow (x+2)(x-1) = 0 \Rightarrow$ the points of intersections are -2 or 1.

The outer radius of a washer $= (2-x)$
~ inner ~ ~ ~ ~ $= x^2$

$$V = \int_{-2}^{1} \pi\left[(2-x)^2 - x^4\right] dx$$

$$= \int_{-2}^{1} \pi\left[(x-2)^2 - x^4\right] dx = \pi\left[\frac{(x-2)^3}{3} - \frac{x^5}{5}\right]_{-2}^{1} =$$

$$\pi\left[\left(-\frac{1}{3} - \frac{1}{5}\right) - \left(\frac{-64}{3} + \frac{32}{5}\right)\right] = \pi\left[\frac{63}{3} - \frac{33}{5}\right]$$

$$= \pi\left[21 - \frac{33}{5}\right] = \frac{105 - 33}{5}\pi = \frac{72}{5}\pi$$

7. Mark each of the following as True or False.

(a) $\frac{d^2}{dx^2}\int_1^x \frac{dt}{t} = \frac{-1}{x^2}$. T because $\frac{d}{dx}\int_1^x \frac{dt}{t} = \frac{1}{x}$

$$\Rightarrow \frac{d^2}{dx^2}\int_1^x \frac{dt}{t} = \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}.$$

(b) $\frac{d}{dx}\int_1^2 \frac{dt}{\sin^2 t + 1} = \frac{1}{\sin^2 x + 1}$. No. F because $\int_1^2 \frac{dt}{\sin^2 t + 1}$ is a constant (real number), hence $\frac{d}{dx}\int_1^2 \frac{dt}{\sin^2 t + 1} = 0$.

(c) $\int_{-1}^1 \frac{1}{t}\, dt = \Big[\ln|x|\Big]_{-1}^{1} = \ln 1 - \ln 1 = 0.$ This is False because $\int_{-1}^1 \frac{1}{t}\, dt$ is not integrable since $f(t) = \frac{1}{t}$ has an infinite discontitity a $\cancel{x}=0$.