King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

Math 102.09 & 14 Exam I Form A Second Semester (052)

ID #: ____________ NAME: Key Solution

Serial # ____________ Section #: ____________

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1. Given $F(x) = \int_0^x \frac{dt}{t^2+9}$, compute

(a) $F(0)$. (b) $F'(0)$. (c) $F'(1)$. (d) $F'(x)$. (e) $F''(x)$.

$F'(x) = \frac{1}{x^2+9} \Rightarrow$ (a) $F(0) = 0$ (b) $F'(0) = \frac{1}{9}$ (c) $F'(1) =$

$F'(x) = \frac{1}{x^2+9}$ (d) $F''(x) = \frac{-2x}{(x^2+9)^2}$.

2. Find f given that $f''(x) = x^2 - x$, $f'(1) = 0$, $f(1) = 2$.

$f'(x) = \frac{x^3}{3} - \frac{x^2}{2} + C$, $f'(1) = 0 \Rightarrow \frac{1}{3} - \frac{1}{2} + C = 0$

$C = \frac{1}{6} \Rightarrow f'(x) = \frac{x^3}{3} - \frac{x^2}{2} + \frac{1}{6} \Rightarrow$

$f(x) = \frac{x^4}{12} - \frac{x^3}{6} + \frac{x}{6} + C_1$, $f(1) = 2 \Rightarrow$

$2 = \frac{1}{12} - \frac{1}{6} + \frac{1}{6} + C_1 \Rightarrow C_1 = \frac{23}{12} \Rightarrow$

$f(x) = \frac{x^4}{12} - \frac{x^3}{6} + \frac{x}{6} + \frac{23}{12}$.

3. Evaluate $\int_2^3 \frac{dx}{x^2-4x+5}$.

$\int_2^3 \frac{dx}{x^2-4x+5} = \int_2^3 \frac{dx}{x^2-4x+4+1} = \int_2^3 \frac{dx}{(x-2)^2+1}$

$= \left[\tan^{-1}(x-2)\right]_2^3 = \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{4} - 0 =$

4. Evaluate $\int_{-1}^{0} x^3(x^2+1)^6\,dx$. We let $u=x^2+1 \Rightarrow du=2x\,dx$, $x^2=u-1$

$\frac{1}{2}\int_{-1}^{0} x^2(x^2+1)^6\,2x\,dx$

$= \frac{1}{2}\int^{1}(u-1)u^6\,du = \frac{1}{2}\int^{1}(u^7-u^6)\,du =$

$\frac{1}{2}\left[\frac{u^8}{8}-\frac{u^7}{7}\right]_2^1 = \frac{1}{2}\left[\left(\frac{1}{8}-\frac{1}{7}\right)-\left(\frac{2^8}{8}-\frac{2^7}{7}\right.\right.$

$= \frac{1}{2}\left[-\frac{1}{56}-2^7\left(\frac{2}{8}-\frac{1}{7}\right)\right] = \frac{1}{2}\left[-\frac{1}{56}-2^7\left(\frac{6}{56}\right)\right]$

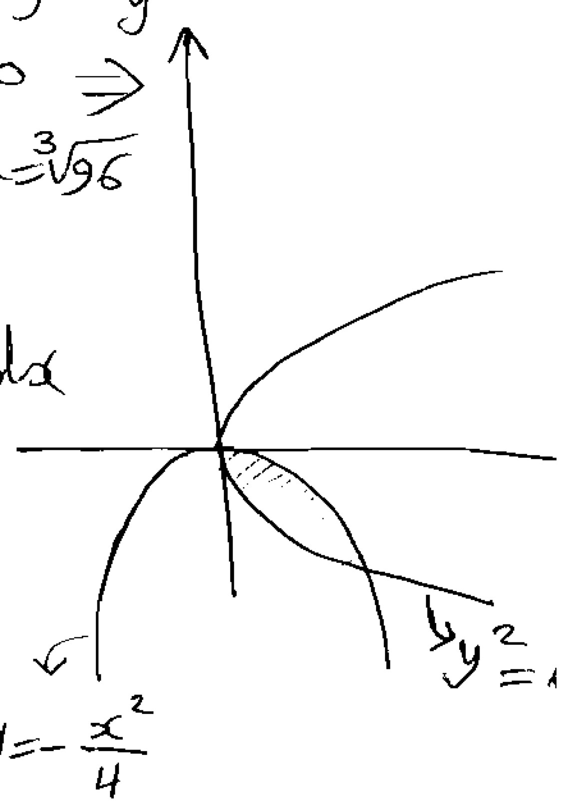
$= -\frac{1}{112}\left[1+2^7\cdot 6\right] = -\frac{769}{112}$

5. Sketch the region bounded by $y^2=6x$ and $x^2+4y=0$ and find its area.

$y^2=6x$, $x^2=-4y \Rightarrow x^4=16y^2$

$\Rightarrow 16y^2=96x \Rightarrow x^4-96x=0 \Rightarrow$

$x(x^3-96)=0 \Rightarrow x=0$ or $x=\sqrt[3]{96}$



The area A needed is

$A=\int_0^{\sqrt[3]{96}}\left[-\frac{x^2}{4}-(-\sqrt{6x})\right]dx$

$=\int_0^{\sqrt[3]{96}}\left[-\frac{x^2}{4}+\sqrt{6}\,x^{\frac{1}{2}}\right]dx$

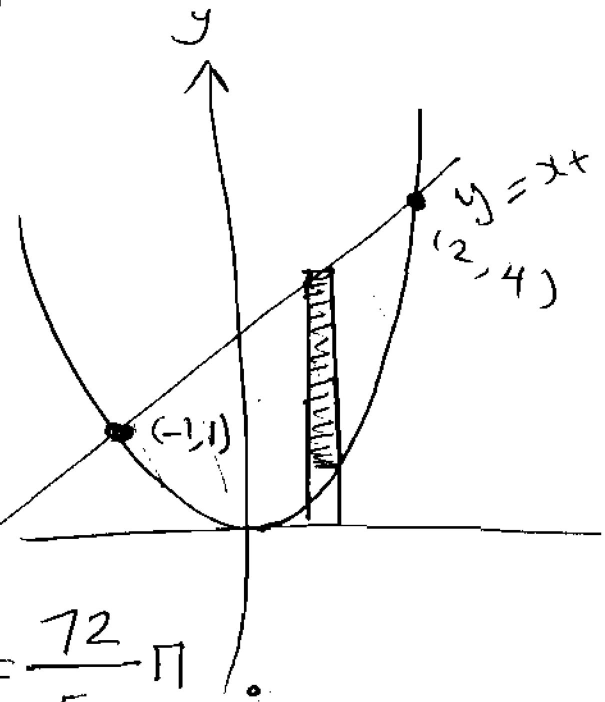
$=\left[-\frac{x^3}{12}+(\sqrt{6})(2)\frac{x^{\frac{3}{2}}}{3}\right]_0^{\sqrt[3]{96}}$

$=\left[\left(-\frac{96}{12}+\frac{2}{3}\sqrt{6}\,\sqrt{96}\right)-0\right] = -8+\frac{2(24)}{3} =$

6. Sketch the region Ω bounded by $y = x^2$ and $y = x+2$ and find the volume [redacted] if Ω is generated about the x-axis.

$x^2 = y = x+2 \Rightarrow x^2 - x - 2 = 0$
$\Rightarrow (x-2)(x+1) = 0 \Rightarrow$
$x = 2$ or $x = -1$

The outer radius $= (x+2)$
The inner radius $= x^2$



$$V = \pi \int_{-1}^{2} \left[(x+2)^2 - (x^2)^2 \right] dx$$

$$= \pi \int_{-1}^{2} \left[x^2 + 4x + 4 - x^4 \right] dx$$

$$= \pi \left[\frac{x^3}{3} + 2x^2 + 4x - \frac{x^5}{5} \right]_{-1}^{2} = \frac{72}{5}\pi .$$

7. Mark each of the following as True or False.

(a) $\frac{d^2}{dx^2} \int_1^x \ln t \, dt = \frac{1}{x}.$ True because $\frac{d}{dx} \int_1^x \ln t \, dt =$ $\ln$

(b) $\frac{d}{dx} \int_1^2 \sin t \, dt = \sin x.$ False because $\int_1^2 \sin t \, dt$ a constant hence $\frac{d}{dx} \int_1^2 \sin t \, dt = 0$

(c) $\int_{-1}^{1} \frac{1}{x^2} \, dx = \left[-\frac{1}{x} \right]_{-1}^{1} = -2.$ False because $f(x) =$ is not integrable on $[-1,1]$ since it has an infinite discontinuity at x