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MATH 301/Term 062/Hw#3(9.7)/

14. Let $\mathbf{F}(x, y, z) = yz\ln(x)\mathbf{i} + (2x - 3yz)\mathbf{j} + xy^2z^3\mathbf{k}$ be a vector function. It is clear that F is defined and differentiable at any point (x, y, z) such that $x > 0$. Moreover we have

$$\begin{aligned} curl\mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz\ln(x) & 2x - 3yz & xy^2z^3 \end{vmatrix} \\ &= (2xyz^3 - (-3y))\mathbf{i} - (y^2z^3 - y\ln(x))\mathbf{j} + (2 - z\ln(x))\mathbf{k} \\ &= (2xyz^3 + 3y)\mathbf{i} + (y\ln(x) - y^2z^3)\mathbf{j} + (2 - z\ln(x))\mathbf{k}. \end{aligned}$$

$$\begin{aligned} div\mathbf{F} &= \nabla.\mathbf{F} = \frac{\partial(yz\ln(x))}{\partial x} + \frac{\partial(2x - 3yz)}{\partial y} + \frac{\partial(xy^2z^3)}{\partial z} \\ &= \frac{yz}{x} - 3z + 3xy^2z^2. \end{aligned}$$

□

24. Let $\mathbf{a} = \alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$ be a constant vector and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. We would like to prove the identity

$$\nabla.[(\mathbf{r}.\mathbf{r})\mathbf{a}] = 2(\mathbf{r}.\mathbf{a}).$$

Indeed we have

$$\begin{aligned} \nabla.[(\mathbf{r}.\mathbf{r})\mathbf{a}] &= \nabla.[\alpha(x^2 + y^2 + z^2)\mathbf{i} + \beta(x^2 + y^2 + z^2)\mathbf{j} + \gamma(x^2 + y^2 + z^2)\mathbf{k}] \\ &= \frac{\partial(\alpha(x^2 + y^2 + z^2))}{\partial x} + \frac{\partial(\alpha(x^2 + y^2 + z^2))}{\partial y} + \frac{\partial(\gamma(x^2 + y^2 + z^2))}{\partial z} \\ &= 2x\alpha + 2y\beta + 2z\gamma = 2(\mathbf{r}.\mathbf{a}). \end{aligned}$$

□

28. Let $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ be a vector function and let f be a real valued function. We would like to prove the identity

$$\nabla \times (f\mathbf{F}) = f(\nabla \times \mathbf{F}) + (\nabla f) \times \mathbf{F}.$$

Indeed we have

$$\begin{aligned}
\nabla \times (f\mathbf{F}) &= = \nabla \times \big(fP\mathbf{i} + fQ\mathbf{j} + fR\mathbf{k}\big) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ fP & fQ & fR \end{vmatrix} \\
&= \Big(\frac{\partial(fR)}{\partial y} - \frac{\partial(fQ)}{\partial z}\Big)\mathbf{i} + \Big(\frac{\partial(fP)}{\partial z} - \frac{\partial(fR)}{\partial x}\Big)\mathbf{j} + \Big(\frac{\partial(fQ)}{\partial x} - \frac{\partial(fP)}{\partial y}\Big)\mathbf{k} \\
&= f\Big(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\Big)\mathbf{i} + f\Big(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\Big)\mathbf{j} + f\Big(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\Big)\mathbf{k} \\
&\quad + \Big(R\frac{\partial f}{\partial y} - Q\frac{\partial f}{\partial z}\Big)\mathbf{i} + \Big(P\frac{\partial f}{\partial z} - R\frac{\partial f}{\partial x}\Big)\mathbf{j} + \Big(Q\frac{\partial f}{\partial x} - P\frac{\partial f}{\partial y}\Big)\mathbf{k} \\
&= f(\nabla \times \mathbf{F}) + (\nabla f) \times \mathbf{F}.
\end{aligned}$$

□