Department of Mathematical Sciences
KFUPM
Term 032

MATH 301-01, 04 / Final Exam/ Duration=3hours

1. Verify the Green's theorem for the line integral $\oint_C ydx + 2xdy$, where $C = C_1 \cup C_2$ and C_1, C_2 are respectively the concentric circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 3$.

2. Let D be the region bounded by the concentric spheres $x^2+y^2+z^2 = a^2$, $x^2+y^2+z^2 = b^2$, $0 < a < b$. Let S be the surface representing the boundary of D. Verify the divergence theorem for D, S and the vector field $\overrightarrow{F} = x\,\overrightarrow{i} + y\,\overrightarrow{j} + z\,\overrightarrow{k}$.

3. Solve the initial-value problem:

$$y'''(t) + y(t) = \delta_\pi(t), \qquad y(0) = y'(0) = y''(0) = 0.$$

4. Find $\mathcal{L}\Big\{e^{3t}\cos^2(t)\mathcal{U}_\pi(t)\Big\}$ and $\mathcal{L}\Big\{t\int_0^t e^{a\tau}\tau^n d\tau\Big\}$, where a is a positive number and n a positive integer.

5. a) Show that the half-range cosine series in $[0,\pi]$ of $f(x) = x^3$ is given by

$$\frac{\pi^3}{4} + \frac{6}{\pi}\sum_{n=1}^{\infty}\Big(\frac{\pi^2(-1)^n}{n^2} + \frac{2(1-(-1)^n)}{n^4}\Big)\cos(nx).$$

b) Assuming that $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$, show that $\displaystyle\sum_{k=0}^{\infty}\frac{1}{(2k+1)^4} = \frac{\pi^4}{96}$.

6. Find the eigenvalues and eigenfunctions of the boundary-value problem:

$$y'' - 2ay' + \lambda y = 0, \quad y(0) = 0,\ y(L) = 0, \quad \text{where } a \text{ and } L \text{ are given positive numbers.}$$

7. Solve the following boundary-value problem :

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial t} & \text{for } 0 < x < \pi,\ t > 0 \\ \dfrac{\partial u}{\partial x}(0,t) = 0, \quad \dfrac{\partial u}{\partial x}(\pi,t) = 0 & \text{for } t > 0 \\ u(x,0) = x^3, \quad \text{for } 0 < x < \pi. \end{cases}$$

8. Use one of the Fourier integral transforms to solve the following boundary-value problem

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2} & \text{for } -\infty < x < \infty,\ t > 0 \\ u(x,0) = e^{-|x|} & \text{for } -\infty < x < \infty, \\ \dfrac{\partial u}{\partial t}(x,0) = 0, & \text{for } -\infty < x < \infty. \end{cases}$$

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