

Department of Mathematical Sciences
KFUPM
Term 032

MATH 301-01, 04/ Exam#2/ Duration=2 Hours

1. Evaluate $\mathcal{L}\{\cos^2(3t)\mathcal{U}_\pi(t)\}$ and $\mathcal{L}\{t^n e^{at}\}$, where n is a positive integer and a a real number.

2. Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2 + s + 1)}\right\}$ and $\mathcal{L}^{-1}\left\{\frac{s}{(s^2 + 1)(s^2 + 4)}\right\}$.

3. Let k and a be given real numbers. Solve the *IVP*:

$$y'(t) + k^2 \int_0^t y(\tau) d\tau = \delta_a(t), \quad y(0) = 1.$$

4. Show that $\cos(3x)$ and $\sin(5x)$ are orthogonal functions on $[-\pi, \pi]$ and calculate the norm of $\sin(5x)$.

5. Let $f(x) = 1$ if $0 < x < \frac{\pi}{2}$ and $f(x) = 0$ if $\frac{\pi}{2} \leq x < \pi$.

Find the half-range cosine and sine series of $f(x)$ in $[0, \pi]$ and show that $\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$.

6. Find the eigenvalues and eigenfunctions of the *BVP*:

$$y''(x) + 2y'(x) + \lambda y(x) = 0, \quad y(0) = 0, \quad y(1) = 0.$$

Put the equation in self-adjoint form and give the orthogonality relation.

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